Long-Term Investment
Asset-Class Based Capital Budgeting

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Oct 2014

Motivation

Most important topic in Corporate Finance?

What do we teach that students need to know?

Capital Budgeting

- Choosing good projects is the most value-important and ubiquitous question.
- Not 1-month projects, but multi-year projects.
- It's our bread and butter
  - Corporate Governance?? Capital Structure??

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Academic Interest

- Let's make sure we get "simple" capital budgeting right!
- Let's make sure it's something our students can apply.

  (Theory is good and useful, but it is not a great applied cost-of-capital estimator.)
- Number of publications in top-5 Journals 2000-2013?

IRR and NPV Logic

- Should you invest their money on behalf of your investors, or should you instead return it?
- Should you demand higher average returns for projects for which similar/equivalent projects are expected to deliver higher returns elsewhere?
- What if the most common models' claims about these other opportunities are wrong?

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What do we **really** teach about Equity Returns?

- Lots of caveats on CAPM/FFM in Fama-French:1997 ...but we still use the models.
- Academic capital-budgeting evidence is based on predictions of 1-mo ($\ll$1 year) ahead stock returns.
  - CAPM fails even on 1-month ahead prediction.
  - Sadly, even FFM may or may not work. (Momentum and book-to-market may work—this is not the FFM!)
- Which corporations really care about the cost of capital for 1-mo (or 1-yr) projects?
  - Interesting projects last 5 years to 100 years
- (Maybe) debt has a lower cost of capital than equity, but WACC is the same (or flat).

Surprising and Not Surprising

- Half of you won't believe **any** evidence, and not abandon the models because you believe they can be useful.
- Half will tell me that existing-models' uselessness was obvious.
- Most will think that other half already shares their views.

So here is what I will "sell" you:

- Some of what I will say will seem obviously true.
- Some of it you will know.
- Some of it will just be repackaged truth—but remember that the Church repeats the gospel many times, too—and it still often does not sink in.
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Key Points

- Equity Premium
  - Widespread (but not universal) misjudgment of hist equity premium.
- Exposure Estimates
  - Universal incorrect prescriptions of long-term exposure estimates

$\Longrightarrow$ Almost-Irrelevance of Equity Return Predictions

- Recap of longer-horizon equilibrium model evidence
  - Not even FFM works, and not even 1-month ahead.

$\Longrightarrow$ Fortunate Almost-Irrelevance of Equity Return Predictions

- Alternative Prescribable Capital-Budgeting Model
  - We have specific alternatives with solid empirical evidence.

Equity Premium for Long-Term Projects

Equity Premium

- We want the forward-looking equity premium.
- Many of us justify an estimate based on backward-looking equity premium.
- ... but many of us have poor memory and/or use the wrong metric to begin with.

It ain't 8%!

- Fun With Figs

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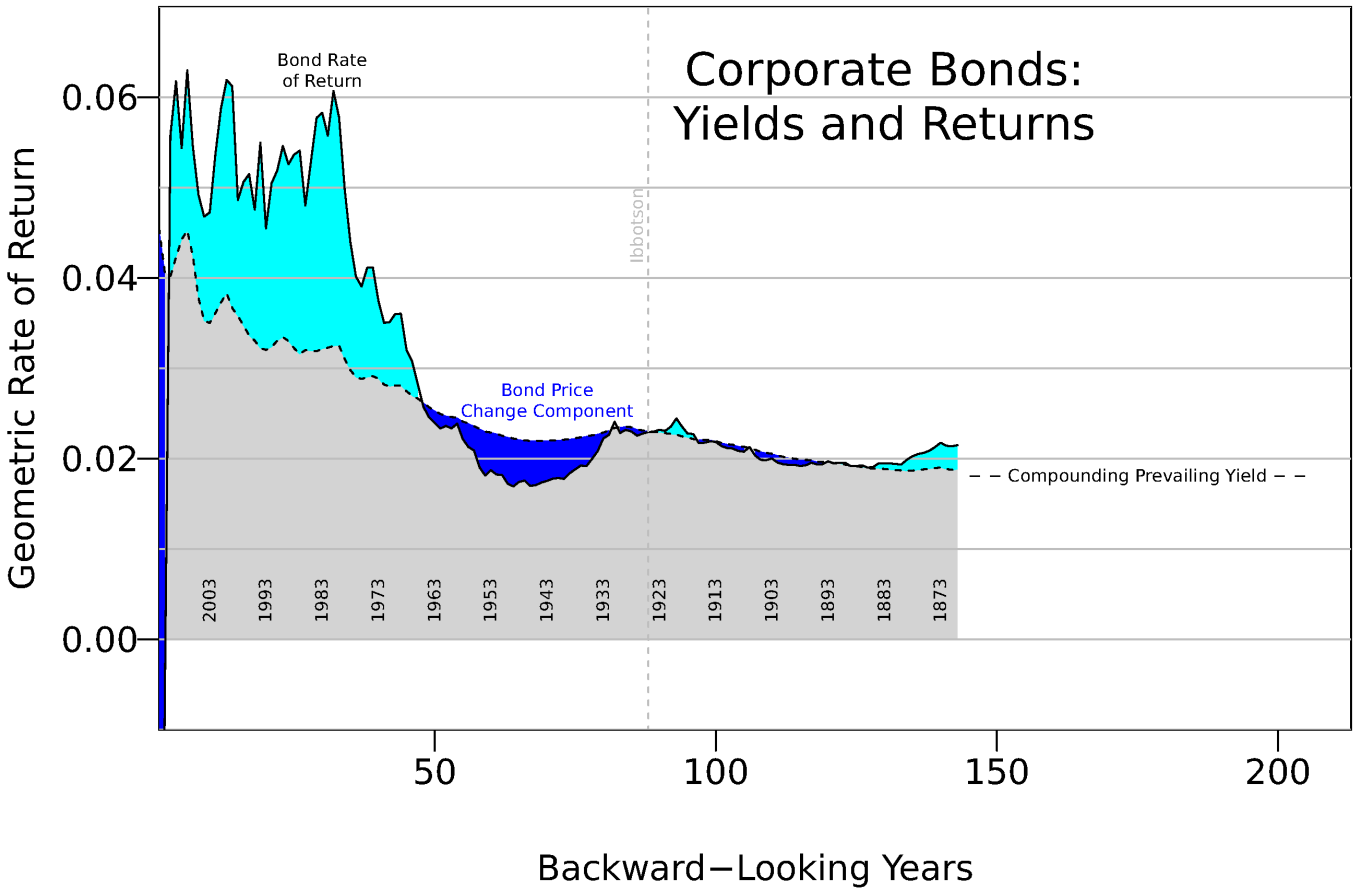
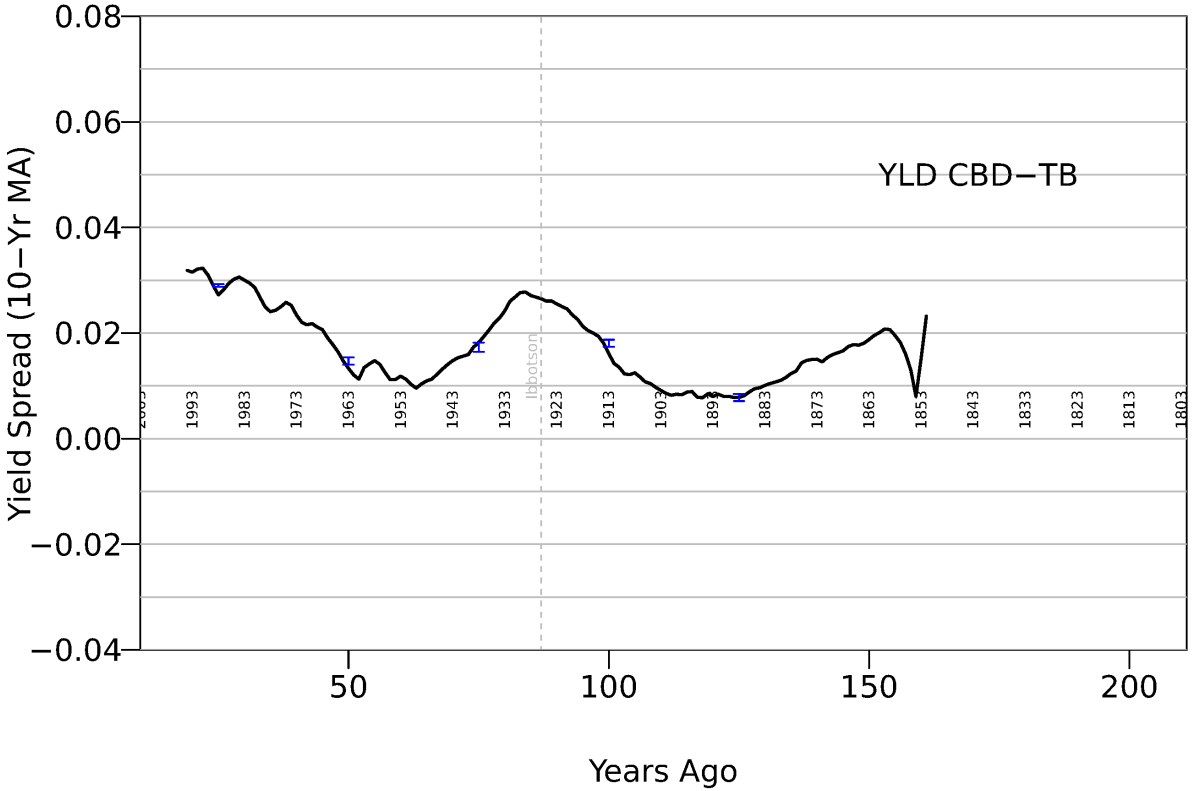
Yields vs Returns on Bonds

Close-To-Tautology:



- Over the very long run, in a stationary equilibrium, long-term bonds have rates of return equal to their yields.

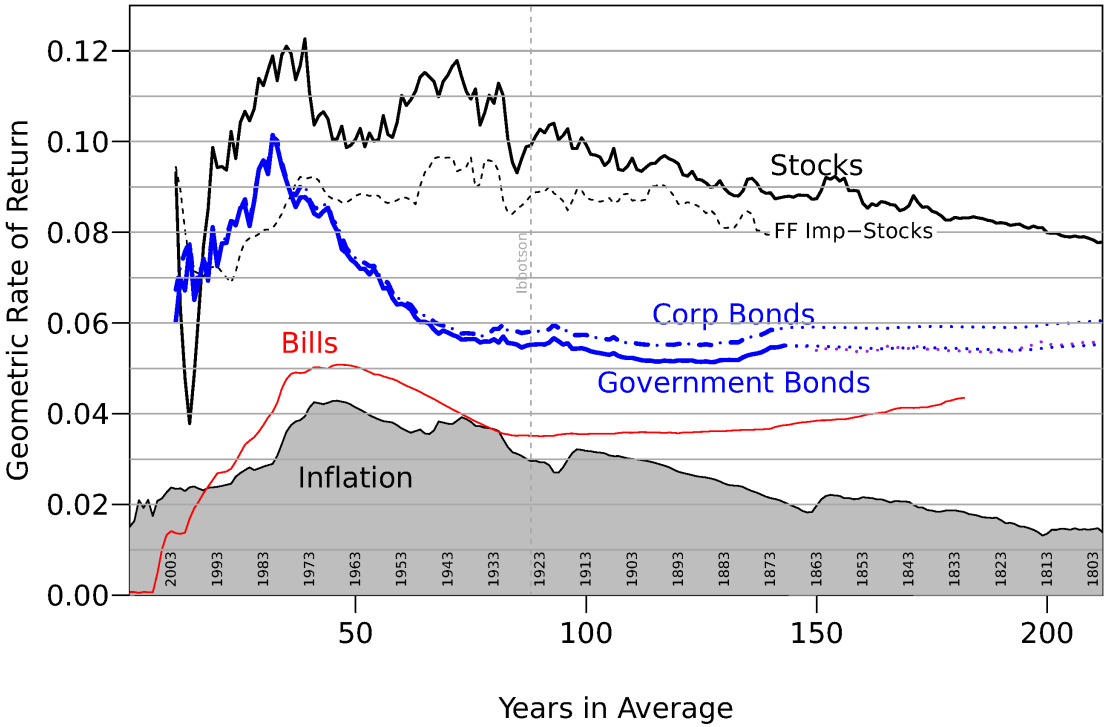
Yield Term Spread



- Bonds tended to yield 2% (0% to 3%) more than bills.
- The obvious: they had higher average yields and higher avg returns.
- They have higher yields in 2014.
- The obvious: maybe not the 2014-bonds, but in the long-run, the bond yield spread will also be the bond return spread.

Geometric Performance To Now

Pick your own:



Long-Horizon Equity Premium Spread (Now=12/2013):

2000-now ≈ 0%	1950–now ≈ 5%
1990–now ≈ 1.5%	1926–now ≈ 4%
1980–now ≈ 2%	1872–now ≈ 3%
1970–now ≈ 2%	1803–now ≈ 2%

(2009 = 26% - (-15%); 2013 = +32% - (-7%) !) LT Eq Prem was lower in 2008/2012

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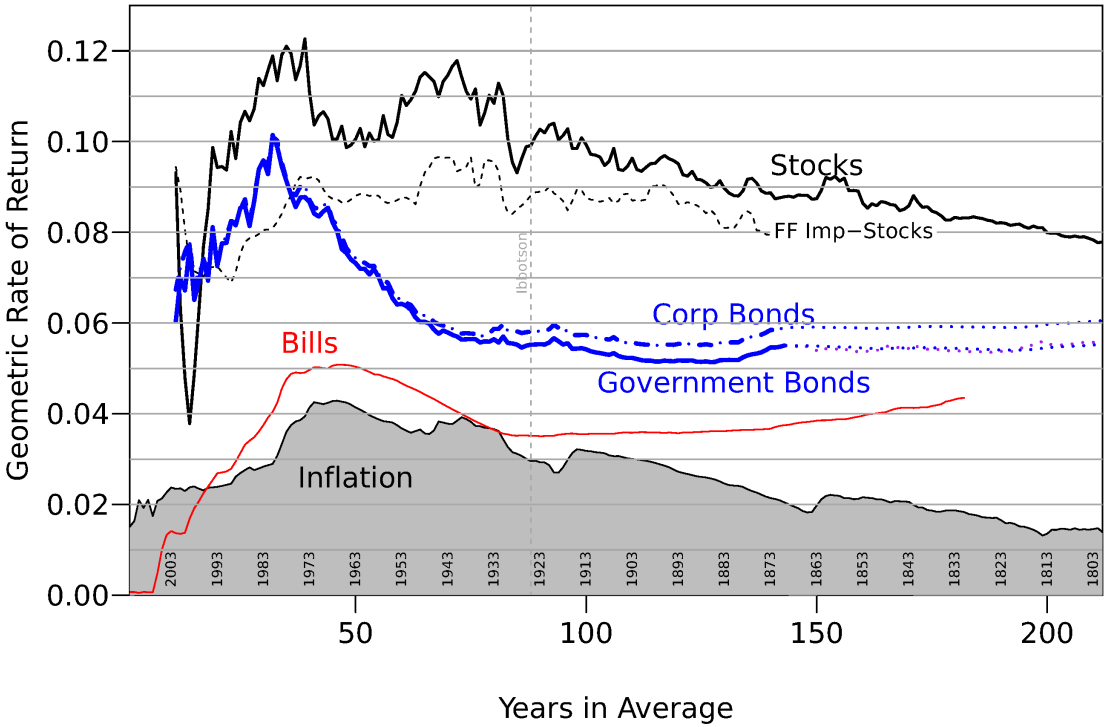

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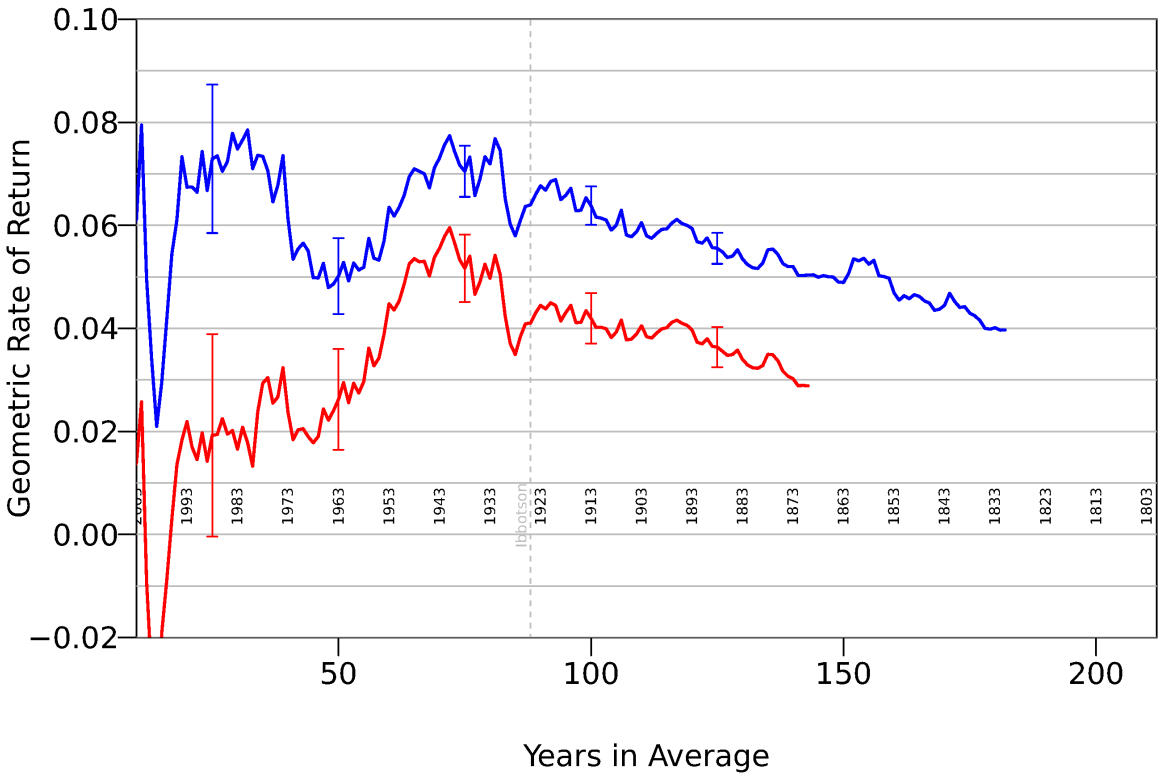
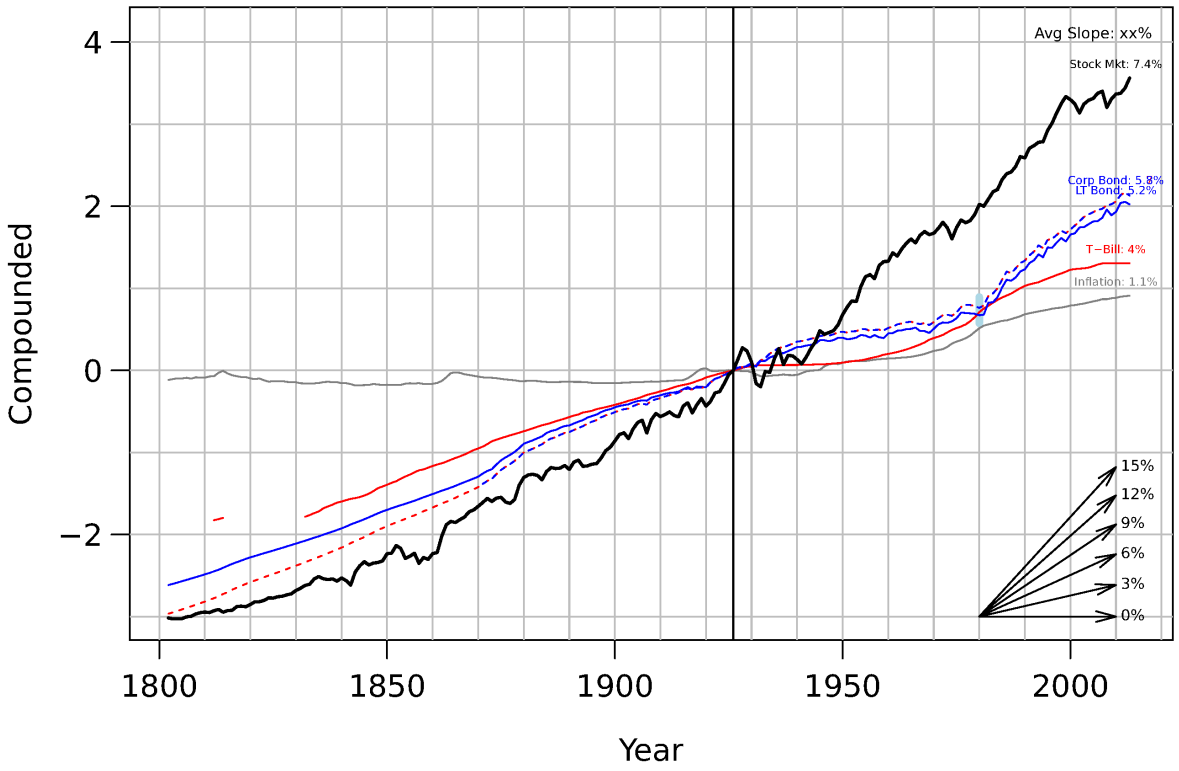
Geometric Equity Premium



- 2% difference between long-term equity premium and short-term equity premium.
- Whatever your choice of equity premium is, it should be about 2% lower for long-term projects than for short-term projects.
- You can't believe in an 8% equity premium with respect to long-term bonds and an 8% equity premium with respect to short-term bills.

Omit Log Plot



- Slopes seem to have shifts.
- Ibbotson period was somewhat unusual.

http://www.ivo-welch.info/professional/goyal-welch/

Historical Inference

Equity Premium

- Principal Data Change: Not lower stock returns nowadays, but higher long-term bond yields nowadays.
- Oft-quoted 6-8% are arithmetic returns from 1926 to 1970 vis-a-vis Treasury bills. R u kidding?
- If based on historical performance, the exp. equity premium relative to LT bonds should be 3% or less. (This is 5% above short-term.)

Me: $< 2\%$.

Non-Historical Inference

It used to be that implied cost of capital (ICCs) were lower than the historical cost of capital.

No longer. Li, Ng, and Swaminathan, JFE2013 extended: Implied Cost of Capital, Based on Analyst Estimates, Oct 2014:

- Relative to Bonds: **6.5%**
- Relative to Bills: 9.7%

I cannot reconcile them. Choose:

- $\approx$ 3% (historical)
- or $\approx$ 6% (ICC).
- I choose $<$ 3%.
- If you choose 6%, you need to worry more about beta than I.

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Long-Term Exposure Estimates

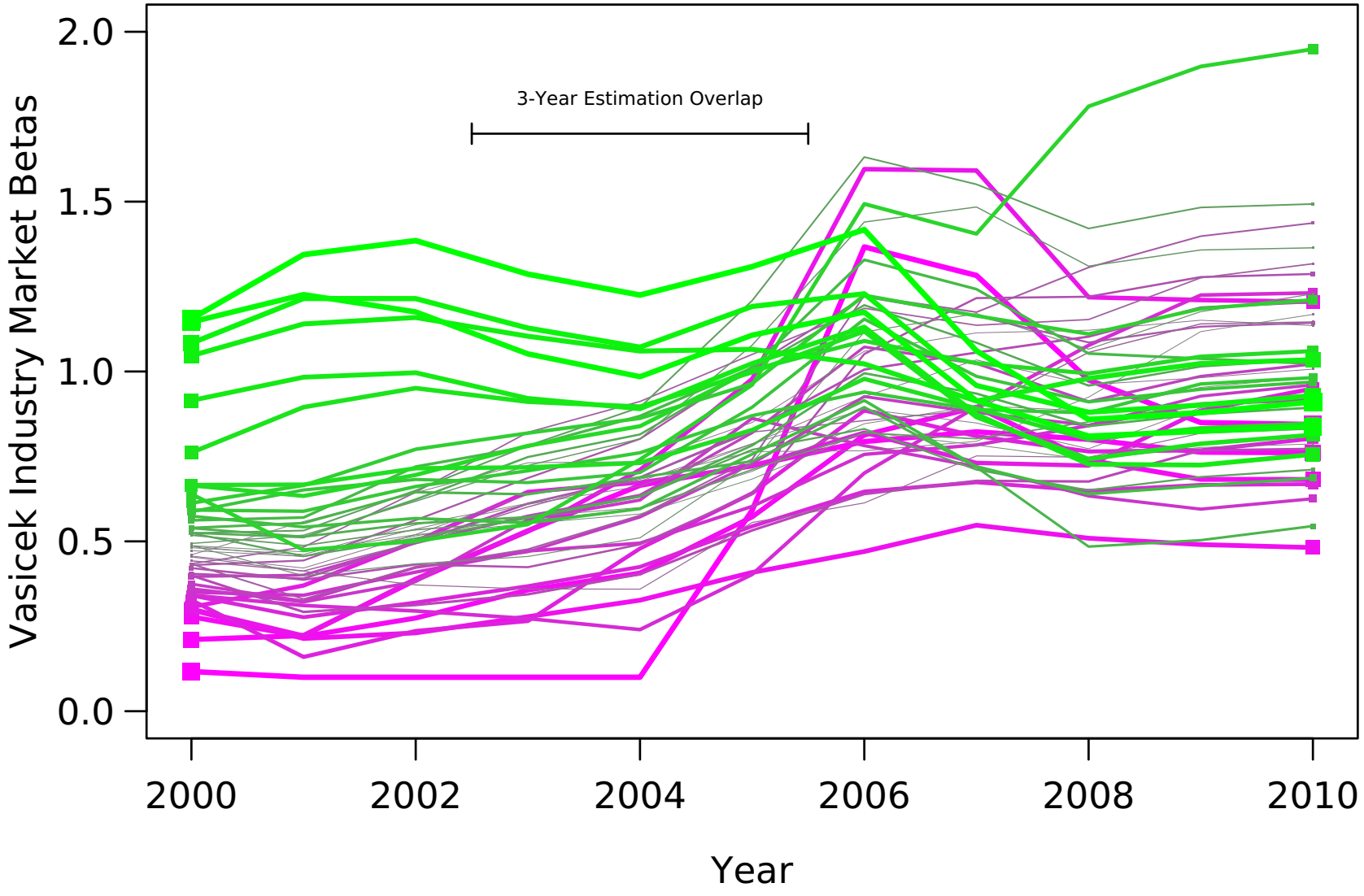
Needed Long-Run Exposure Adjustments

Even if you are a believer, your models' estimates/loadings do not have much long-term stability. (Stability is necessary, but not sufficient. Stability is *not* a tough model criterion. Needed in long-term applications.)

I will show you that today's beta estimates cannot be used for cash flows in 5-10 years.

- This is *after* Bayesian Vasicek exposure shrinking.
- CAPM estimates, say, 5% E(R) difference in cc today
  $\Longrightarrow$ optimally use= 2% E(R) diff for 5-year's CFs (Car)
  $\Longrightarrow$ optimally use= 1% E(R) diff for 20-year's CFs (Building)
  $\Longrightarrow$ optimally use= 0% E(R) diff for 50-year's CFs (Land)
- Is this a good use of your research money? (Gaming?)

Beta Stability of Equity (Not Assets)



(10-year autocoef for 49 industries is about 0.4.)

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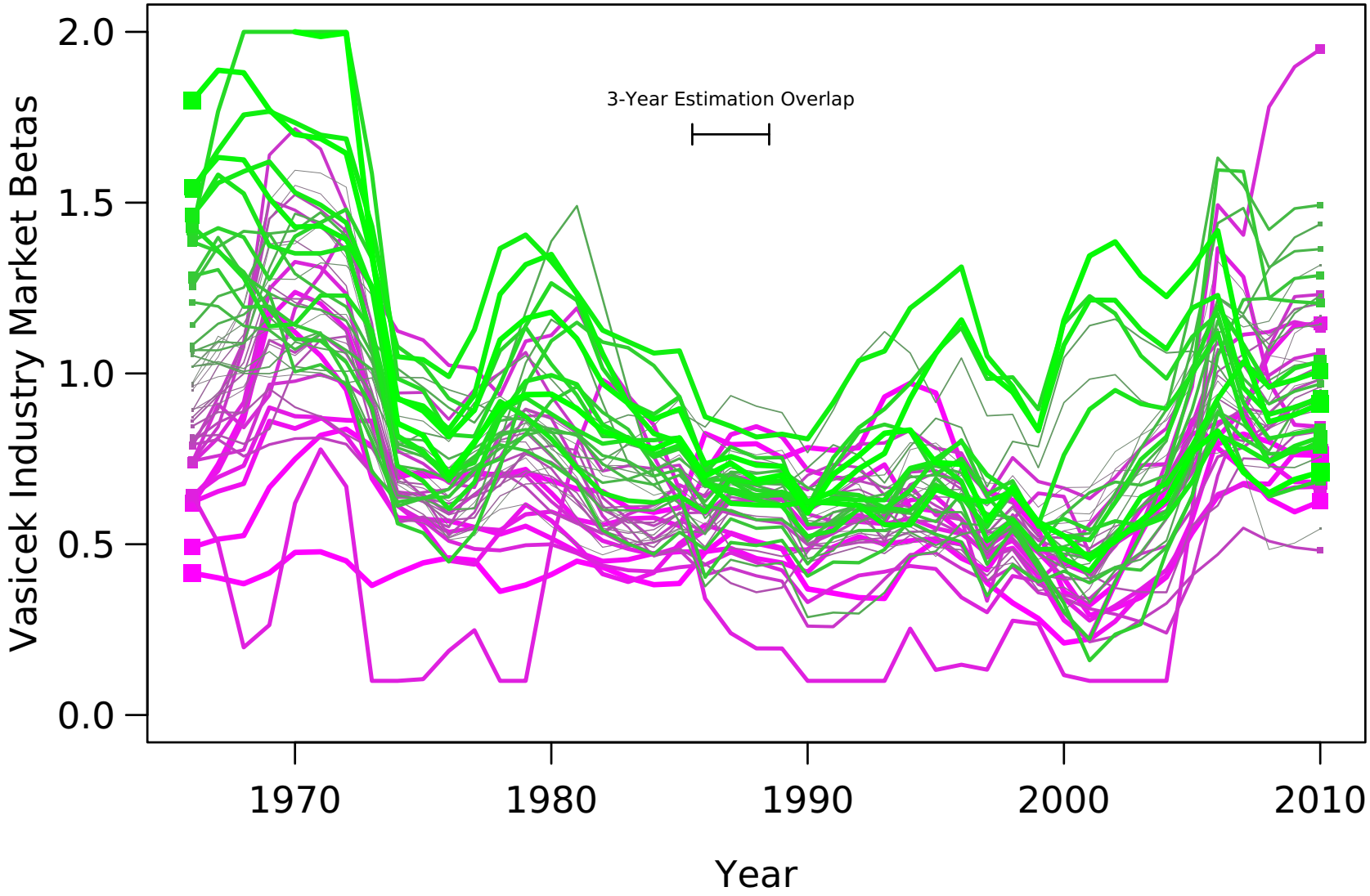
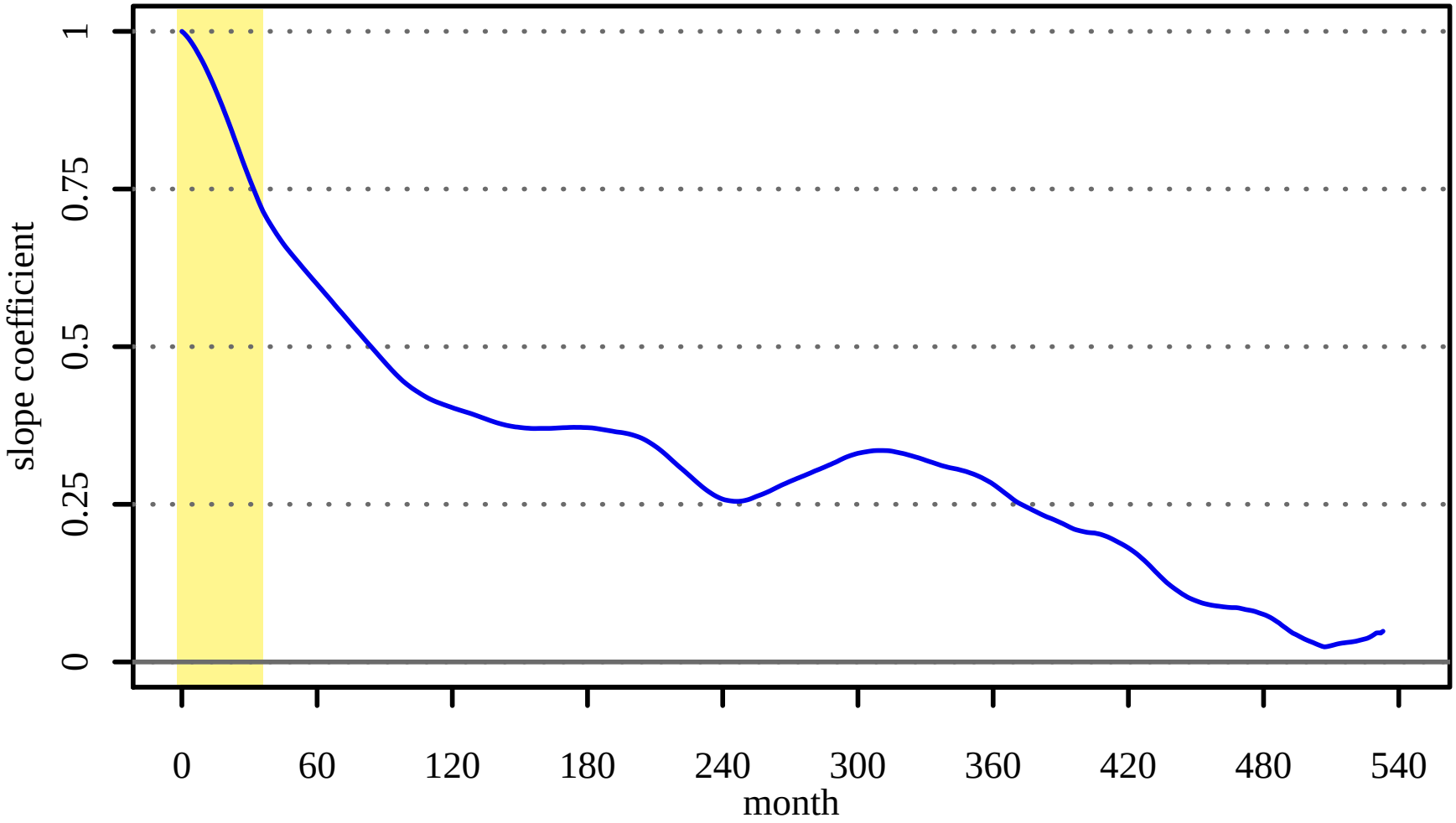


(50-year autocoef for 49 industries is about 0.)
(FFM loadings are similarly or more unstable.)

X-Sectional Correlation of Industry ER over Time



Warning: final data points are based on very few regressions.

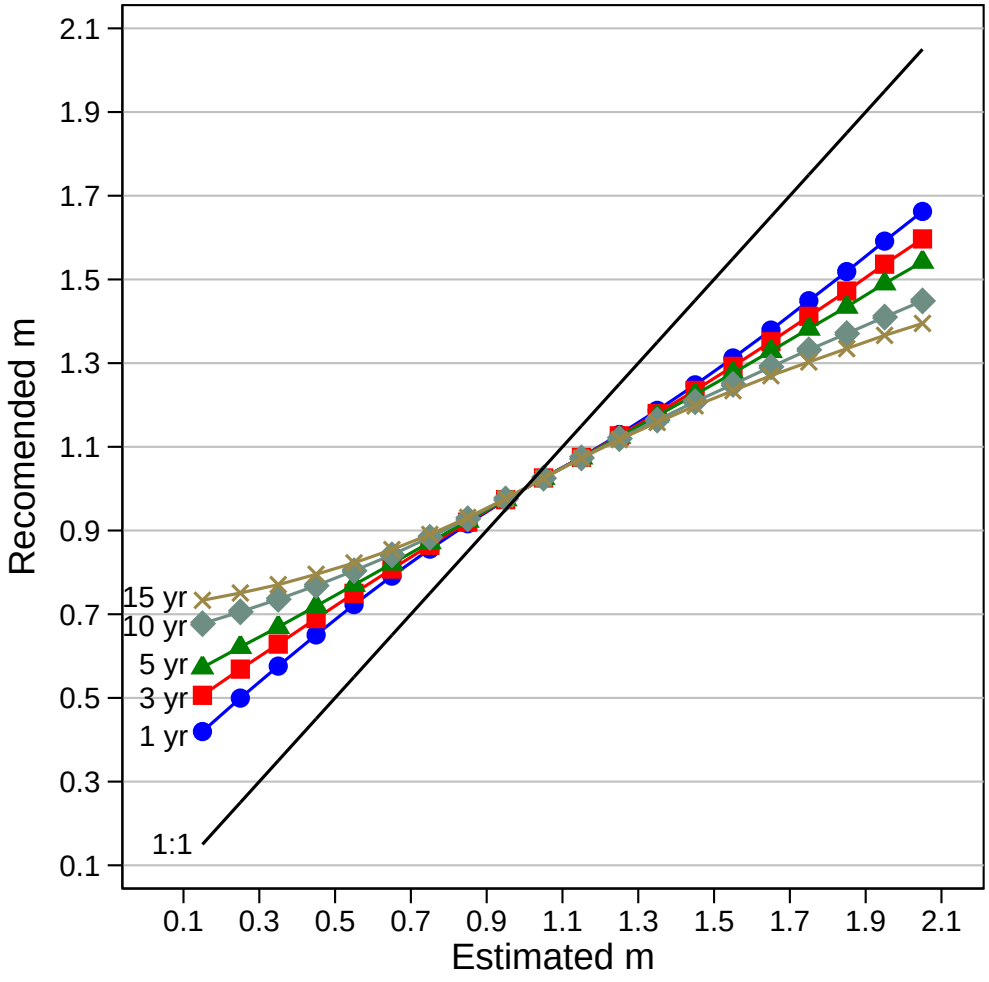
Optimal Weight on Vasicek

- Assume CAPM is true. Simulate World (know true ER).
  - Match beta reversion: $m_t \approx 0.01 \times 1 + 0.99 \times m_{t-1} + e$
  - Match E(M), sd(M), sd(e). sd[E(R)].
  - No LR industry own means. just long-run but temp moves.
- Estimate Vasicek beta and cost of capital.
- Find best θ weighted Vasicek beta / E(r) and "1.1" that minimizes MSE difference to true E(r).

  Double shrinkage:
  - Shrinkage / Vasicek says put some weight on 1.1, some weight on your own beta.
  - With autocorr of beta, we need to shrink more.

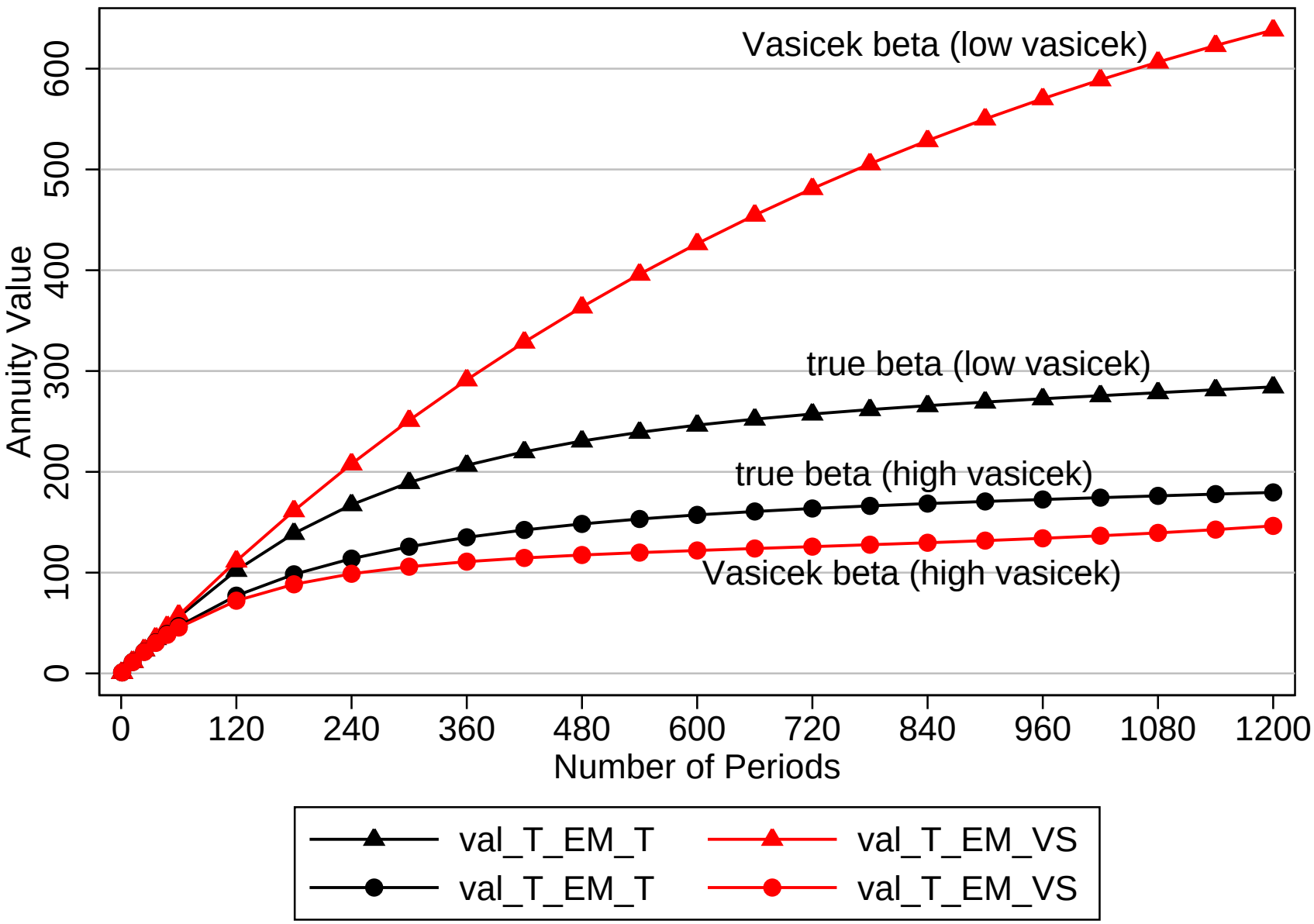
How should you double-shrink Beta?

What shrinkage tells you, vs what you should be using:



X-axis is already the Vasicek shrunk beta!

Annuities Value Effects



Model Empirical Validity

Model Evidence

- Preceding was internal model validity. It did not look at actual "other project" opportunity costs—actual rates of return delivered.
- So, did the models have any predictive ex-ante power for what other projects with similar model riskiness actually delivered ex-post?
- Q: You know the 1-mo evidence. What do you think the 10-yr evidence is?

Predict future actual returns with your model returns (not with model ingredient factors).

$$r_i = \gamma_0 + \gamma_1 \times E(R_i) + \text{noise}$$

Aggregate over time. Doesn't matter much.

Quick Summary of Presentation Figs

We test reasonable model implementation and use; we do not test the model per se:

- Only 49 Industries. (Indiv. firms = worse. no IPOs, survival)
- 1962–2010. (21,683 stocks / 2.1m firm-months)
- Vasicek betas, daily data, 5 year windows. FFM=MV.
- 30-50 year prevailing premia estimates.
- Use models to calculate expected rates of return.
- How do model X="expected rates of return" *predict* future Y=E(r) or future actual Y=r? Ideally, $\hat{\gamma}_1 = 1$. Useful model if $\hat{\gamma}_1 > 0$.
- Xsect Q: Always out-of-sample, Fama-Macbeth like.
- All standard errors are from **placebo**: randomize returns across firms/industries on same date. Keeps irregular data matrix intact. We do not randomize factor premia—if we destroyed them, NULL would look even better.

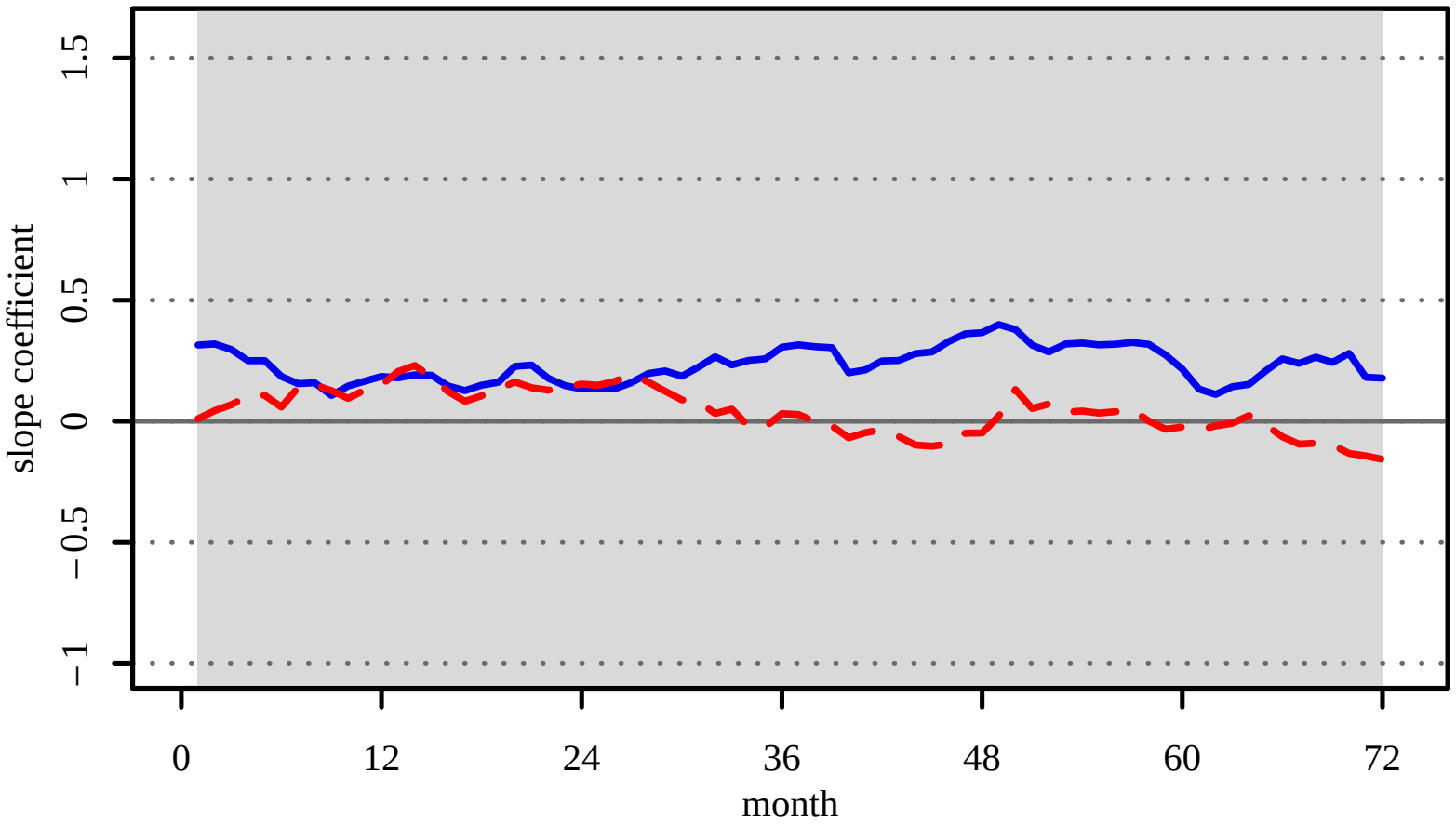
Sort of a best-use-case scenario

Presentation omits MANY robustness checks.

Start

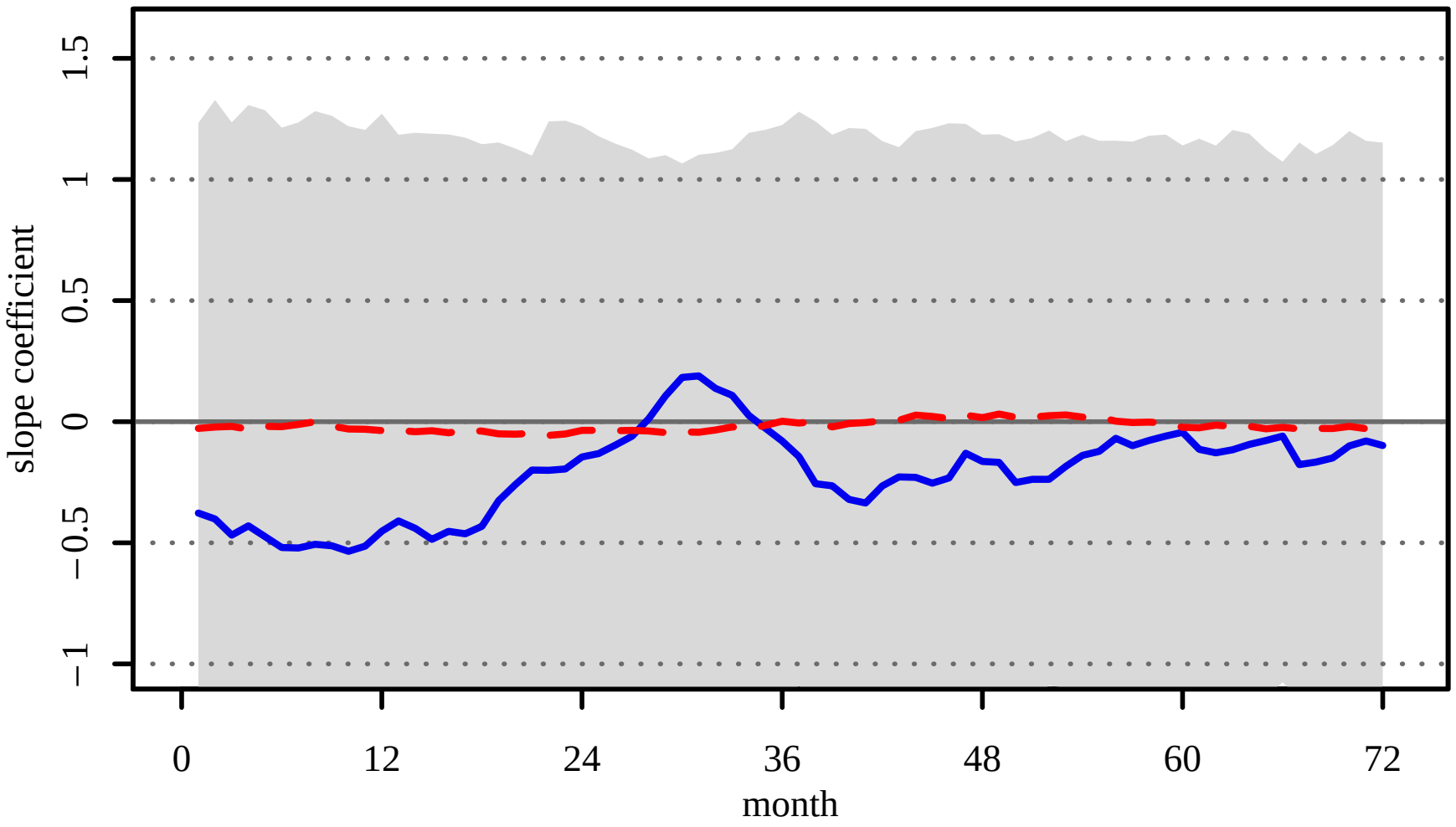
(Easier to show than to explain. Equities Only! Not Unlevered!)

CAPM – Marginal Returns

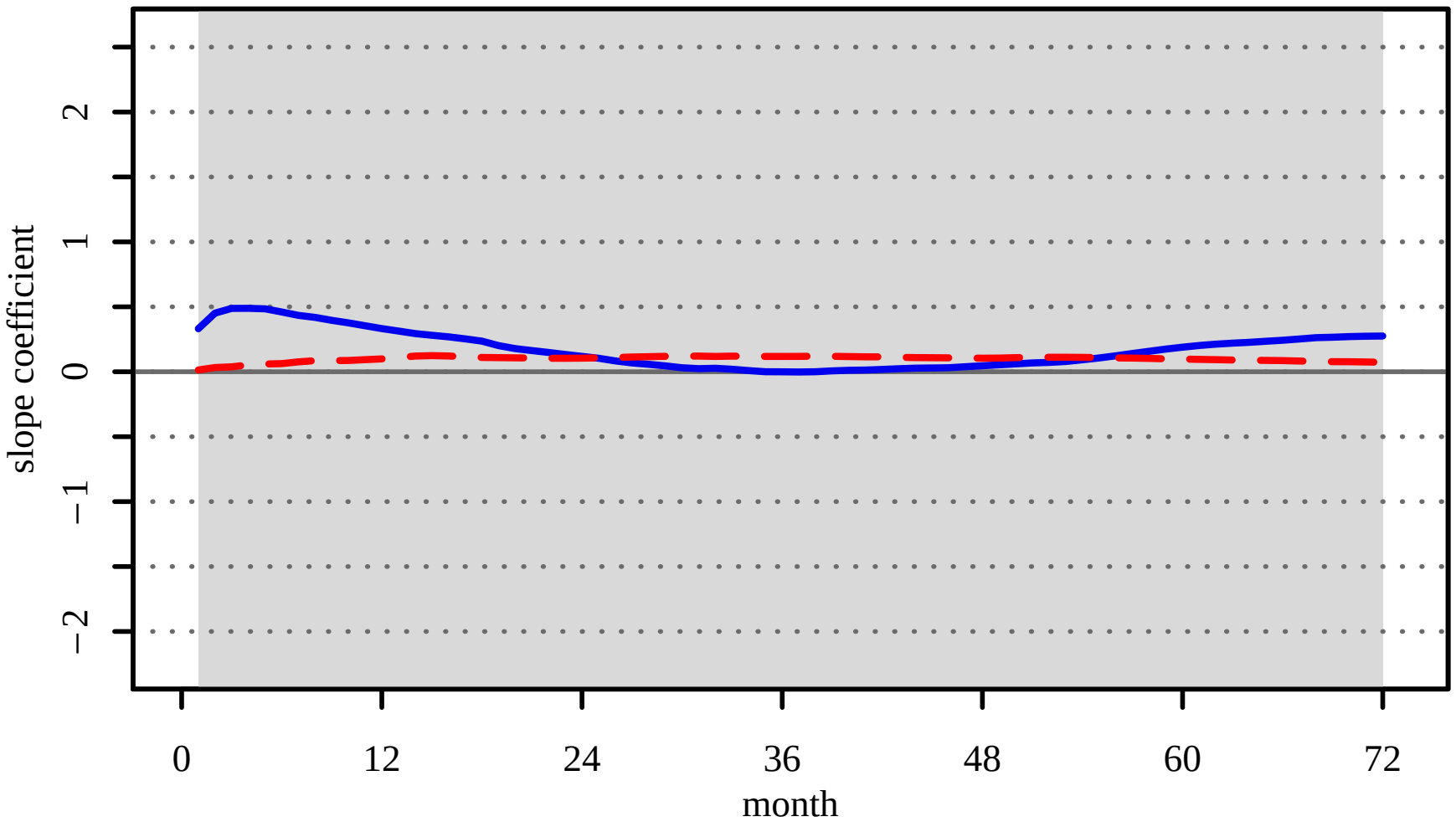


IAW: Stop and Explain Graph.

FFM – Marginal Returns



CAPM – Compound Returns



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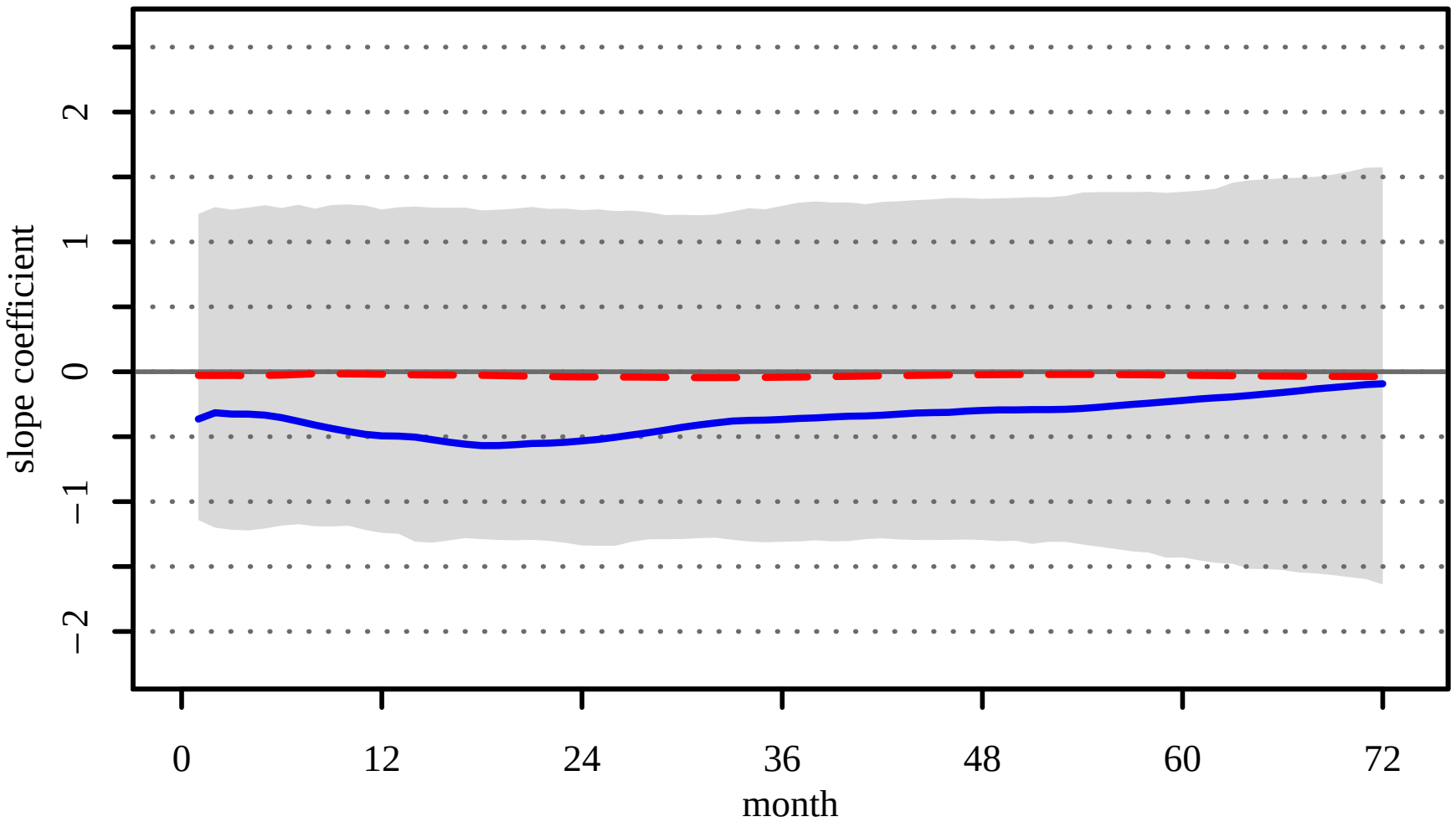

Does the FFM hold??

- Sorry, no. Not even over short intervals.
- Some factors inside FFM have worked: B-M and Momentum.
- Recent papers suggest only about 600 different factors that have worked...in-sample. After publication, about 51% work, the other 49% fail.
- Lewellen (CFR 2015) may have a better characteristics-based predictive model on **shorter** horizons...but it is not online and/or forecasts are not available to corporate execs. Thus, it is de-facto secret to them.

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Risk

Did more risky firms offer higher average (not promised!) returns?

Sadly no

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Eric Falkenstein Video

Financial Genius

Conclusions?!

- Neither the FFM nor the CAPM model had actual-return forecasting power over long-horizons, either. Not even a close call.
- As benchmark providers for what expected returns projects should have provided over the long term, both models have utterly failed "use test" in the past. Not even a close call.
- They had no hope of making meaningful **differential** cost of capital predictions.
- They had no hope of success.
  - Not in the sense of "could the models be true" but in the sense "could the models be useful?" No EIV.
- Confess: Were your priors that the models could give you good estimates over longer horizons??
- Confess: What are you teaching to your students (besides caveats)?

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Leverage

Did highly levered firms offer higher average returns?

Sadly no. Not really. See below.

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What Works?

Now What?

It takes a model to beat a model.

What should we teach? Would can we teach?

- Asset-Class Differential Cost of Capital
- Term-Spread Differential Cost of Capital

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Asset-Based Capital Budgeting

- We are interested in asset betas, not equity betas: $E(R_A) = w_E \times E(R_E) + w_D \times E(R_D)$.
- For whatever reason (imperfect markets?), all equities seem to offer similar long-term average returns.
- If your $E(R_D) < E(R_E)$, **and** you can predict own future D/E, then you can predict future **asset** cost-of-capital.
- Leverage ratios are often predictable and/or stable.
- It's a standard CorpFin (not AssPrc) approach. Assign one cost of capital to equity. Assign one cost of capital to debt. (Debt capacity can be useful.) Take wght avg.
- Corporate income tax deduction may well be most of the reason why bonds end up being cheaper corporate financing than stocks.

ABC: Asset-Based Capital Budgeting

For long-term standard corporate projects:

- Assume $\beta_{\text{Equity}} \approx 1$.
- Use (tax-adj) cost of debt capital; for AAA or secured, maybe promised $\approx$ expected
- Assess your planned/intended project debt-ratio.
- Use

$$\textbf{ABC} \Rightarrow \; E(R_A) = \hat{w}_E \times (\underbrace{3\% + 3\%}_{\text{term-adjusted}} \cdot 1) + (1 - \hat{w}_E) \times \underbrace{E(R_D) \times (1 - \tau)}_{\text{Imperfect-Mkt-adjusted}}$$

- Possibly worry about cost of capital of NFL implicit in project.

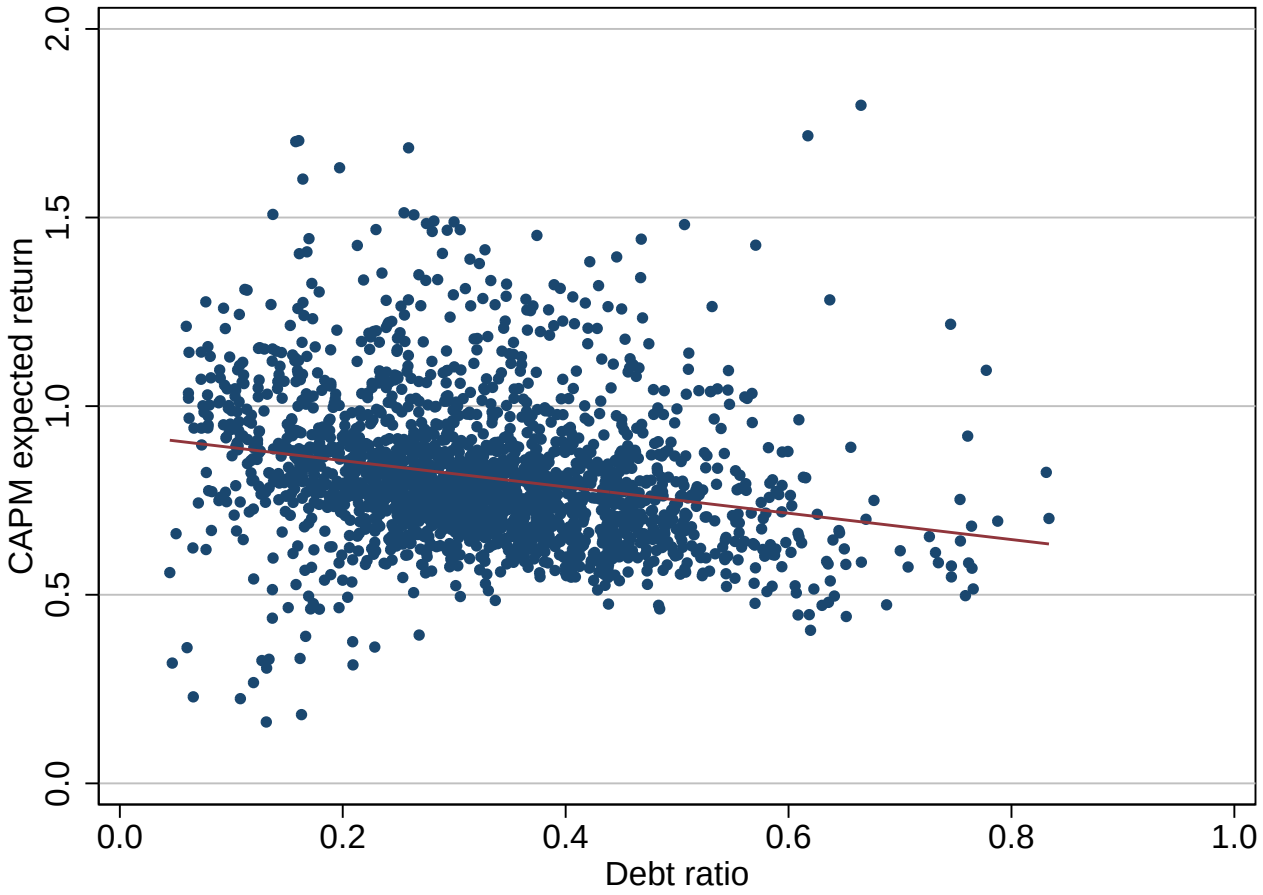
And spend your time worrying about E(CF) instead. You think we don't have enough to teach here??

Mistakes?

How bad are ABC errors relative to true CAPM/FFM?

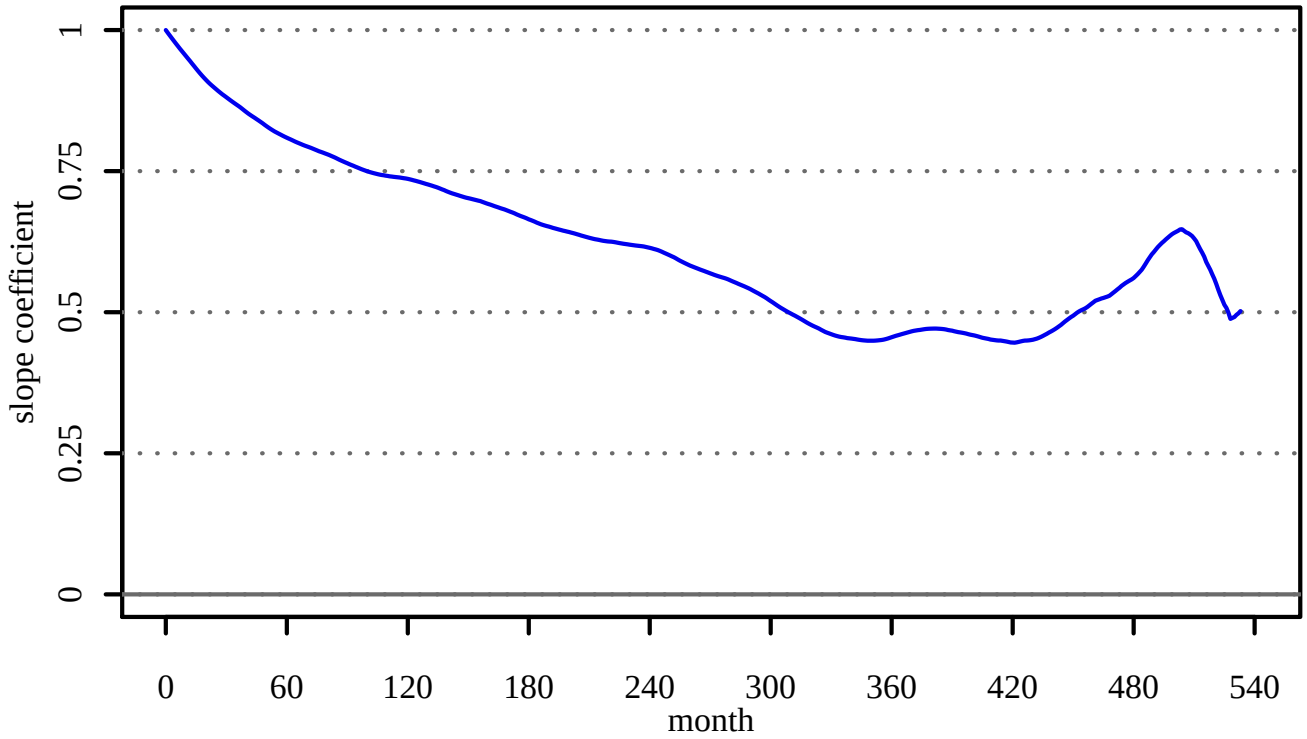
- Don't use this model for (short-term) bond pricing or for 99% levered companies. Use this model for normal firms/projects.
- Leverage ameliorates further asset-beta errors. Errors in $E(R_E)$ typically map into lower $E(R_A)$ errors. High leverage, high $E(R_E)$ errors are mult. by $1-w_D$.
  - High-leverage same-asset-beta firms should have had high w_E, $E(R_E)$, and $E(R_D)$.
  - Empirical Evidence:
    high LR = high $E(R_E)$? see next pg.
    high LR = high $E(R_D)$? maybe. see Altman etc.

Leverage Ratios and Model Equity Expected Rates



(model exp, not act; industry or firms; not pos, even when firm-type controlled. it ain't strong. don't think negative—think "almost nothing.")

Leverage Ratios and Future Leverage Ratios



(but debt may well be your decision variable, so you don't need this)
(even with controls for firm type, the relation remains very weak.)

Is Corporate Debt Really Cheaper than Equity?

We think so, but even this is **not** 100% clear. Net Annual Rate of Return (not Yield): Market – Corporate AAA Bond:

Period	Geo Net	Std Err
–1947	1%	(s.e. $\approx$ 2%)
1950–1980	8%	(s.e. $\approx$ 3%)
1980–2013	1.5%	(s.e. $\approx$ 3%)
1872–2013	2.7%	(s.e. $\approx$ 1.5%).

- Was the (**pre-tax**) cost of corporate AAA bond-financing really lower?
  - A 3% difference is barely statistically significant, and it's *mostly* 1950–1980.
  - Otherwise, "stocks minus long-term AAA bonds" was under 2%.
  - You need $\approx$ 400 (1,600) years to tell at 2% (1%).
- **Important:** The after-tax cost of corporate AAA bonds (and short-term AAA bills) does seem meaningfully lower.

Fortunately, like w_D, this can be a firm-specific CFO judgment call. (From the inside, in an imperfect market, quoted yields may even be your expected cost of debt.)

Natural Consequences

Optimal behavior is similar to some imperfect-market corporate theories, but ABC is more pragmatic and less specialized.

- Value debt-financed projects (like buildings) more highly than equity-financed projects (like R&D).
- Don't put equity money into cash. The presumed reduction in equity betas which reduces the cost of capital is *not* there. Holding cash is not worth it.
- Take projects until the marginal cost of debt is equal to the marginal cost of equity and the marginal return on projects.
- Conjecture—firm may incur sudden sharp increase in the cost of debt and equity when "overlevered."

Advantages

- Most Important: (Academic) Integrity.
- Truth in Advertising.
- Not priors=faith-based capital budgeting.
- Lots of tough problems become much easier.
  - E.g., real options turn from real hard into real simple problems.
  - E.g., tax shelters are simple now. APV and WACC yield the same results.
  - E.g., behavioral finance may be easier to understand.
- Less distraction with unimportant details.

- No claims to short-term AP. ABC is not all the answers.

Managerial Advantages

- Focus more on time and less on risk adjustments.
- Focus more on expected cash flows—which is where the focus should be!
- Focus more on failure probabilities (cash flows).
  - Higher expected rates of return for high-failure projects based on an asset-pricing model is the wrong crutch.
  - Maybe helped by a volatility-based E(R) model?
- Easier (=cheaper) to use same cost of equity capital for all projects.
- Less gaming.

Unhelpful Appendix

(1) Model for #1A: Dynamic-Beta CAPM

$$m_{i,t}^{true} \overset{iid}{\sim} N(\mu_m, \sigma_m) \qquad t = -35 \qquad m_{i,t}^{true} = \mu_{dm} + \rho_{dm} m_{i,t-1}^{true} + \varepsilon_{dm} t \qquad = -34, \cdots, 180 \tag{1}$$

$$\varepsilon_{dm} \overset{iid}{\sim} N(0, \sigma_{dm}^2) \qquad M_t \overset{iid}{\sim} N(\mu_M, \sigma_M^2) t \qquad = -599, \cdots, 0 \tag{2}$$

$$MP \sim N(\mu_M, \sigma_{MP}^2) \tag{3}$$

$$r_{i,t} = r_f + m_{i,t}^{true} M_t + \varepsilon_{i,t} \qquad t = -35, \cdots, 0 \qquad \varepsilon_i \overset{iid}{\sim} N(0, \sigma_E^2) \tag{4}$$

$$E(r_{i,t})^{true} = r_f + m_{i,t}^{true} \mu_M \qquad t = 0, 1, \cdots, 180 \tag{5}$$

Manager estimates her loading over 36 periods.

$$r_{i,t} - rf = \alpha_i + m_i^{est} M_t \qquad -35 \leqslant t \leqslant 0 \tag{6}$$

Manager chooses her cost of capital by weighting her own estimated cost of capital and the cross sectional mean,

$$COND = E(r_i)^{est} = r_f + m_i^{est} MP \tag{7}$$

$$UNCO = r_f + \mu_m MP \tag{8}$$

We find the optimal weight by simulating the model and solving

$$min_{w_t} E\left[\left(w_t UNCO + (1 - w_t) COND - E(r_{i,t})^{true}\right)^2\right] \qquad t = 0, \cdots, 180 \tag{9}$$

Note that the dynamics of $m_{i,t}$ in equation 1 can be represented as

$$m_{i,t}^{true} = \theta K + (1 - \theta) m_{i,t-1}^{true} + \varepsilon_{dm} \tag{10}$$

with $\theta = 1 - \rho_{dm}$, $K = \frac{\mu_{dm}}{1 - \rho_{dm}}$.

Estimation

Direct estimation We set $\mu_m, \sigma_m, \sigma_E, \mu_M, \sigma_M$ and r_f equal to the corresponding population moments. See table pop dynamics.

Calibration We set μ_{dm}, ρ_{dm} and σ_{dm} to fit the population moments in tables 49 ind. The calibration process for the 49 industries simulations is as follows:

- We construct a panel, size 49 industries and 108 periods ($t = -35, \cdots, 72$), of true market loadings. We draw $t = -35$ loadings for the 49 industries from a normal distribution with mean μ_m and std σ_m (see table **??**). True loadings evolve over the additional 107 periods according to equation 1.
- We draw a ts of factor (M) realizations from a normal distribution with mean μ_M and variance σ_M. (See table **??**.)
- We construct a panel of realized returns using the ts of factor realizations, the panel of true loadings and σ_E from table **??**.
- We construct a panel ***estimated*** loadings using the realized returns and the factor realizations.
- We construct find expected returns using the estimated loadings and market premium drawn from a normal distribution with mean μ_M and variance σ_{MP}.
- We repeat this process 1000 times and present the means of the collected moments in table tables **??**.

Dynamic model parametrization, CAPM, direct estimation

model parameter	sample	value	source
μ_M		0.458	XMKT 600 month ending at 2010/12
σ_M		4.525	XMKT 600 month ending at 2010/12
σ_{MP}		0.185	standard error of μ_M
rf		0.049	rf 36 month ending at 2010/12
σ_E	49 industries	4.797	average of error term std in loading estimation re
μ_m	49 industries	1.115	mean XMKT loading
σ_m	49 industries	0.309	std XMKT loading
σ_E	all CRSP	13.030	average of error term std in loading estimation re
μ_m	all CRSP	1.097	mean XMKT loading
σ_m	all CRSP	0.779	std XMKT loading

Dynamic model parametrization, matched moments, 49 industries

statistic	49 industries sample		calibration results*		
	value	s.e.	$t = 0$	$t = 36$	$t = 72$
μ_m	1.115	0.009	1.113	1.113	1.111
σ_m	0.309	0.004	0.343	0.340	0.340
$corr(m_t, m_{t+1})$	0.987	0.001	0.992		
$corr(m_t, m_{t+36})$	0.560	0.008	0.577		
$corr(m_t, m_{t+72})$	0.444	0.009	0.423		
$std(E(ret)^{est})$	0.188	0.003	0.158	0.158	0.156

* Chosen calibrated parameters are $\mu_{\text{dm}} = 0.01, \rho_{\text{dm}} = 0.991, \sigma_{\text{dm}} = 0.04$.
Population moments are ts averages of the monthly data 1966/07 to 2010/12.
Population market loadings were estimated using 36 historical month
Population expected returns were constructed using constant risk free rate (0.049) and 600 months running average of XMKT.

CRSP, betas below 0.5 at t=0 , moments in population vs simulations

statistic	All CRSP sample			calibration results*		
	t=0	t=36	t=72	t=0	t=36	t=72
μ_m	0.173	0.657	0.680	0.006	0.585	0.754
$s.e.(\mu_m)$	0.005	0.008	0.008	0.003	0.004	0.003
σ_m	0.325	0.623	0.620	0.409	0.678	0.670
$s.e.(\sigma_m)$	0.007	0.008	0.008	0.002	0.005	0.004
$std(E(ret)^{est})$	0.222	0.403	0.387	0.196	0.306	0.296
$s.e.(std(E(ret)^{est}))$	0.005	0.006	0.005	0.008	0.012	0.014
$corr(m_t, m_{t+1})$	0.924			0.965		
$corr(m_t, m_{t+36})$	-0.003			0.288		
$corr(m_t, m_{t+72})$	0.021			0.192		

CRSP, betas above 1.5 at t=0 , moments in population vs simulations

statistic	All CRSP sample			calibration results*		
	t=0	t=36	t=72	t=0	t=36	t=72
μ_m	2.112	1.477	1.357	2.045	1.537	1.389
$s.e.(\mu_m)$	0.010	0.012	0.012	0.003	0.003	0.002
σ_m	0.591	0.789	0.793	0.440	0.680	0.673
$s.e.(\sigma_m)$	0.009	0.012	0.012	0.002	0.005	0.004
$std(E(ret)^{est})$	0.387	0.510	0.498	0.211	0.307	0.298
$s.e.(std(E(ret)^{est}))$	0.006	0.007	0.007	0.008	0.012	0.014
$corr(m_t, m_{t+1})$	0.950			0.969		
$corr(m_t, m_{t+36})$	0.152			0.306		
$corr(m_t, m_{t+72})$	0.122			0.206		

(2) List of Omitted Variations

- Firms rather than Industries. We do not have project data. Firms with IPOs. (Problem: Survival.)
- Variations in factor premia assessments. Full-sample ex-post. 50-year. 30-year.
- No-adjustment beta. Blume-adjustment. ML adjustment. Dimson beta. Conditional Vasicek beta (size, leverage, book-market).
- Beta = 5 years, daily. 5-years monthly (worse). excess vs. raw regressions.
- Equal-weighted vs. value-weighted factor portfolios
- Industry portfolios, equal-weighted vs value-weighted. 49 vs. more.
- Forecast compound returns with and without volatility adjustment. (1/2 sigma-squared)
- Forecast discount factors.
- Model expected return calculation:

$$E_t[r_i] = r_{f,t} + \hat{\beta}_{i,M} \cdot \overline{XMK}_t \ ,$$

$$E_t[r_i] = r_{f,t} + \hat{\beta}_{i,M} \cdot \overline{XMK}_t + \hat{\beta}_{i,S} \cdot \overline{SMB}_t + \hat{\beta}_{i,H} \cdot \overline{HML}_t \ .$$

- Placebo-adjustment for overlap. Non-overlap. Omitted Model Factors.
- Worry about worry—placebo seems most robust.
- HML model, instead of FFM model.