Market-Beta

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April 2019

with Yaron Levi: “Market-Beta and Downside Risk”
solo: “Model-Based Winsorizing Estimators: Simpler Estimators For Market Beta”

Notice to PhD Students

My papers are intended to teach you how to (not) commit suicide on the job market.

- The one with Yaron is not making friends.
- The solo is too simple.
- ...and neither is about new data, Kenya, and/or clever quasi-experimental identification.

But I think both papers contain important and useful empirical findings, so I hope not to waste your time.

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Motivation

Why still bother with “boring” old market-beta?

- Market-beta is interesting even w/o CAPM
  - Measure of risk contribution to diversified portfolios.
  - Hedging against bear markets
  - Down-Beta Theories (as in Ang+ or Lettau+)
  - Betting against Beta (as in Frazzini-Pedersen)
  - Pragmatic: used in regulation, etc.
- How should we estimate beta?
  - #2 offers new, easy, and superior estimator.

Down-Beta (with Yaron)

Three connected parts:

1. All-days market-beta is a good measure of stocks' hedging aspects for bear and crash markets.
2. A strong critique of downside beta in equities
   (Ang-Chen-Xing (2006), > 200 WoS > 800 Google)
   - Critique = Perspective. All results are replicable.
   - Definition: Down-beta is on days when $R_M < 0$.
3. A mild critique of downside beta in asset classes
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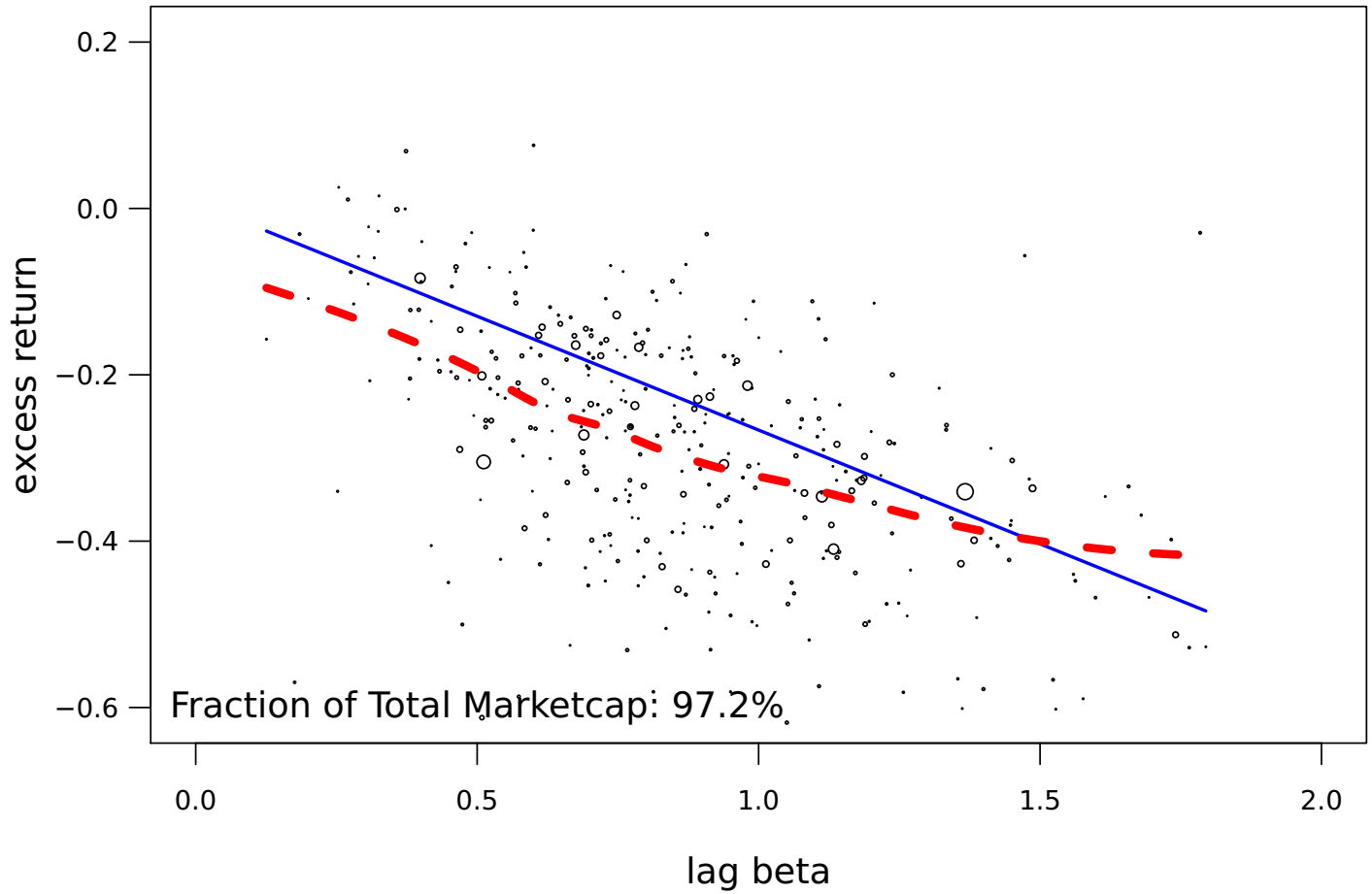
Part 1: Plain Beta As Hedge Metric

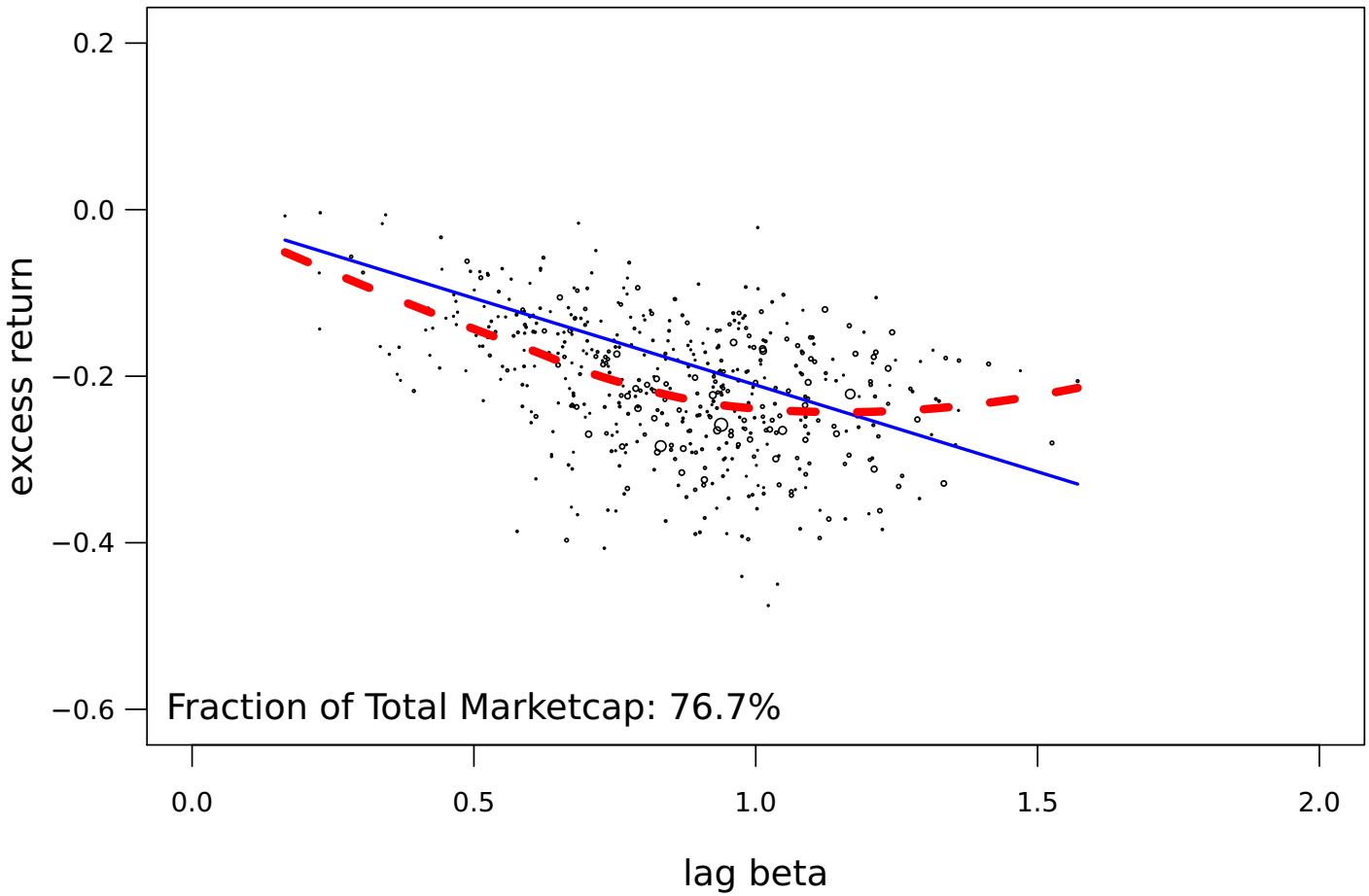
- Lots of detail (in the paper).
  - Daily-return “all-days” betas. OLS and/or others.
- Result: Plain=all-days beta is a good exposure measure also for down and crash markets.
- Will just show you the 3 extreme periods.
  - Betas are estimated ex-ante (all-days)
  - Market performance is realized in-time.
  - Select= Crash. Stocks. X-Axis is beta. Y-axis is returns.

1929: Oct 28, Oct 29, Nov 06

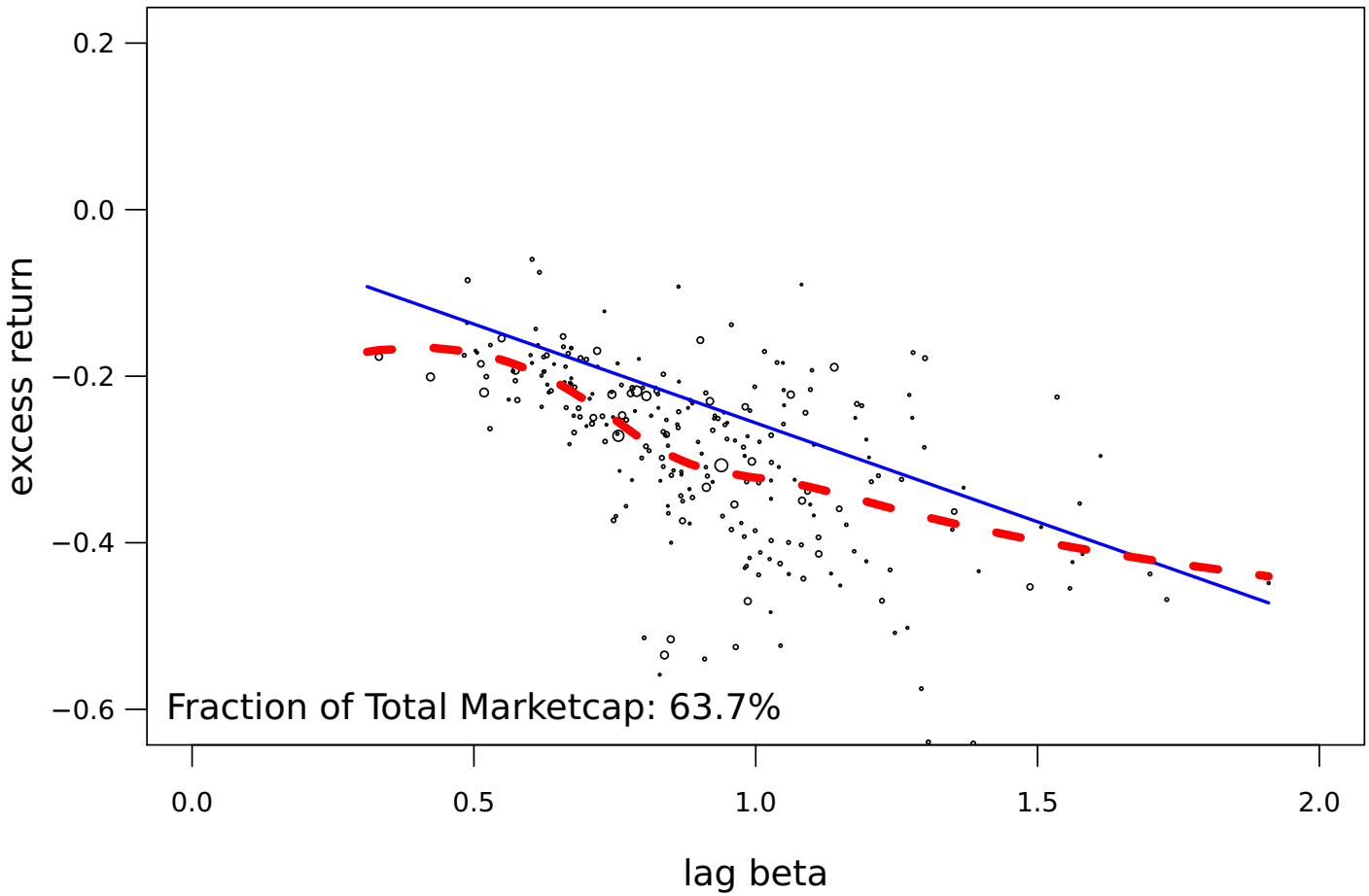


Blue = ex-ante OLS beta predicted slope
Red = loess realized smoothed fit ex-ante

1987: Oct 16, Oct 19



2008: Oct 7, 9, 15 + Dec 1



Part 2: Down-beta in Equities

- Can we improve (down-market) hedging?
- Estimate beta only on market down-days: $\hat{b}_y^-$
  - Estimate beta on market up-days $\hat{b}_y^+$, too.
- **Is down-beta the relevant risk measure?**
  - Roy (1952), Markowitz (1959), etc.
- Is there a premium for down-beta bearing?
- **Most Prominent: Ang-Chen-Xing (2006)**

pause

especially at CU!

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ACX Innovations

- Earlier tests used monthly betas and formed pfios that destroyed variation in $\hat{b}_y$.
  - E.g., they may have sorted on $\hat{b}_y$.
  - it is better to work with individual stocks.
- ACX sometimes use set of low-volatility stocks.
  - LV = Low-Volatility.
  - LV is ex-ante pre-identified. Good idea.

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ACX Synopsis

1. Down-betas can forecast future down-betas.
2. Simultaneous Down-Beta Return Association.
   - The realized down-beta correlates strongly with **contemporaneous** average returns.
   - And this is also **not** mechanical. ✓
3. Some Down-Beta Future Return Evidence.
   - Down-betas can also predict quintile pfio returns.
   - (Plain, BkMkt+Sz+UMD adjusted)
4. Some significance in GMM on 25 FF pfios.

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Still Relevant?

- ACX remains highly influential.
  - >200 Web of Science, >800 Google Scholar
  - Influence is not declining.
  - Will become “home run” paper.

- We critique ACX’s inference, **but**
  - All ACX results are replicable.
  - There are no mistakes.
  - Our paper “only” revisits interpretation of evidence.

Descriptive Statistics

Low-Volatility (LV) Subsample:

		Mean	Sd	#days
All-days-Beta	$\hat{b}_y$	0.67	0.54	253
Down-Beta	$\hat{b}_y^-$	0.72	0.62	116
Up-Beta	$\hat{b}_y^+$	0.61	0.64	132
Abs(Down – Up)	$\lvert\hat{b}_y^- - \hat{b}_y^+\rvert$	0.40	0.43	

Calendar Year Betas. 240k firm-years. LV 1927-2016.

1. Down-betas can forecast future down-betas

- Of course, we all agree that investors care not about past but about future down-beta.
- ACXT7: down-beta can predict future down-beta:

$$\hat{b}^{-}_{y} \approx \mathbf{0.56} \cdot \hat{b}^{-}_{y-1} + c + e, \qquad R^2 \approx 30\%$$

ACXT7 is basically right!

N $\approx$ 240k. i subscripts on $\hat{b}^{-}_{y}$ and e. Panel or FM. se is tiny. estimates.

- But if you care about $\hat{\bar{b}}_y$, can you do better?
- All-days beta $\hat{b}_{y-1}$ always has about twice as many days for estimation as down-beta $\hat{\bar{b}}_{y-1}$,
- ...and it has more X-axis support,
- ...but if $\hat{\bar{b}}_y$ (process) is truly different, down-beta could predict itself better,
- ...or not.
- **Empirically easy to investigate.**
  - Not shown: our conclusions are **very** robust.

- But if you care about $\hat{b}_{y}^{-}$, can you do better?
- All-days beta $\hat{b}_{y\_1}$ always has about twice as many days for estimation as down-beta $\hat{b}_{y\_1}^{-}$,
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- ACX: Predict $\hat{b}^{-}_{y}$ with lagged down-beta:

$$\hat{b}^{-}_{y} \approx \mathbf{0.56} \cdot \hat{b}^{-}_{y-1} + c + e, \qquad R^2 \approx 30\%$$

- LW: Predict $\hat{b}^{-}_{y}$ with lagged all-days betas:

$$\hat{b}^{-}_{y} \approx \mathbf{0.72} \cdot \hat{b}_{y-1} + c + e \qquad R^2 \approx \mathbf{40\%}$$

$$\hat{b}^{-}_{y} \approx \mathbf{0.74} \cdot \hat{b}_{y-1}$$

$$-0.07 \cdot \hat{b}^{+}_{y-1} + \mathbf{0.05} \cdot \hat{b}^{-}_{y-1} + c + e \quad R^2 \approx \mathbf{40\%}$$

$N \approx 240k$. i subscripts on $\hat{b}^{-}_{y}$ and e. Panel or FM. se is tiny.

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- **If you care about the future down-beta, then forecast it with all-days beta, not with itself.**
  - Or shrink $\hat{b}^{-}_{y-1}$ away to almost nada.
- ...because

$$(\Delta_y \equiv)\ \hat{b}^{-}_{y} - \hat{b}^{+}_{y} \approx c + 0.087 \cdot (\hat{b}^{-}_{y-1} - \hat{b}^{+}_{y-1})$$

Most Δ_y is just estimation noise.

(PS: It is this noisy realized betas that is also the one used in ACX part 1. It must have huge EIV. (Not shown:) some is even harder-to-estimate time-variation in Δ.)

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Above was down-beta prediction.

Below is stock-return explanation/prediction.

2. Simultan Down-Beta vs Return

Philosophical Points, Ex-Post Ω

- First half of ACX uses ex-post simultaneous down-betas to explain rates of return.
- It is defensible that representative investors know stocks' **true** down-betas better than us.
  - But must be very smart aggregators for pricing!
- But it seems implausible that they know the **realized** down-betas (from the very same returns being predicted!), and/or any other single year.
  - At least, use many years [–4 to +4 = no results].

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T2: Fama-Macbeth, Simul Realized

$$r_{yi} = \gamma_0 + \gamma_1 \cdot \hat{b}^{=}_{yi} + \gamma_2 \cdot \hat{b}^{+}_{yi} + \ldots$$

Beta		ACX RFS Simultans $\hat{b}_y$	Replic $\hat{b}_y$
$\hat{b}^{=}$		0.062	0.088
	(T)	(+6.0)	(+6.1)
$\hat{b}^{+}$		0.020	0.002
	(T)	+2.3	+0.2
Sample		ACX 1963-2001	ACX

(Strong positive for $\hat{b}^{=}$ only if betas are estimated simultaneous (or one future year). $\hat{b}^{=}$ is not positive in longer windows around returns. Not shown, 90% of power is from all-days beta, too. Controls were included, but are not reported. About 500k obs/2.2m obs.)

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$\hat{b}^{=}$	0.062	0.088	–0.009
(T)	(+6.0)	(+6.1)	(–1.6)
$\hat{b}^{+}$	0.020	0.002	–0.005
(T)	+2.3	+0.2	(–0.8)
Sample	ACX 1963-2001	ACX	ACX 1963-01

(Strong positive for $\hat{b}^{=}$ only if betas are estimated simultaneous (or one future year). $\hat{b}^{=}$ is not positive in longer windows around returns. Not shown, 90% of power is from all-days beta, too. Controls were included, but are not reported. About 500k obs/2.2m obs.)

T2: Fama-Macbeth, Simul Realized

$$r_{yi} = \gamma_0 + \gamma_1 \cdot \hat{b}^{-}_{yi} + \gamma_2 \cdot \hat{b}^{+}_{yi} + \ldots$$

Beta	ACX RFS Simultans $\hat{b}_y$	Replic Simultans $\hat{b}_y$	Ex-Ante $\hat{b}_{y-1}$	
$\hat{b}^{-}$	0.062	0.088	–0.009	–0.022
(T)	(+6.0)	(+6.1)	(–1.6)	(–3.5)
$\hat{b}^{+}$	0.020	0.002	–0.005	–0.020
(T)	+2.3	+0.2	(–0.8)	(–3.6)
Sample	ACX	ACX	ACX	Extd
	1963-2001		1963-01	1927-16

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Fama-Macbeth Gammas on

- 63-01: **Realized** down-betas $\hat{b}_y^- \xrightarrow{+}$ returns. (0.08)
- 63-01: "Placebo"
  Ex-post (**plain**) betas $\hat{b}_y \xrightarrow{+}$ returns. (0.18)
- 63-01: Ex-post competing effect:
  $\hat{b}_y = 0.21^{***}$ $\hat{b}_y^- = \mathbf{0.03}^{**}$ $\hat{b}_y^+ \approx -0.04$
- 63-01: **Ex-ante** any betas: $\xrightarrow{-}$ returns.
- 63-01: **Windowed 4yr** betas: $\xrightarrow{-}$ returns.
- **1963-2016**: $\approx$ 63-01.

Defend Ex-Post Realized Beta?

- Fama: all AP tests are eqbm model and Ω.
  - Judgment call: ex-post info seems better in IV regressions, agent-specific consumption, etc.

- Ex-post info could resolve many pricing mysteries.
- Most important, FM all-days beta $\rightarrow$ stock returns:

with	FM Gamma	(T-stat)
... Ex-Ante Betas	–0.3%/year	(–0.22)
... Contemp Betas	+8.4%/year	(+3.84)

and 8.4% is even underestimated due to EIV. See original FM multi-sort, etc.

Above was ACX ex-post down-beta evidence (ACX T2-T5).

Below is ACX ex-ante down-beta evidence (ACX T8-T10).

... and GMM (ACX T6)

3. Down-Beta Future Return Evidence

ACX Specification:

- Quintile test pfios based on down-betas.
  - Short: Downbeta ≈ 0.2.
  - Long: Downbeta ≈ 1.9.
- Zero-Investment Portfolio Tests
  - Jensen-Black-Scholes (1972), Fama-French (1993).
- non-LV and LV sets.

ACX Tables 8-10

Lagged beta predicts future monthly stock returns:

(not reported) $\hat{b}_{y-1}$	0.19	1.89
(not reported) $\hat{b}_{y}$	0.60	1.38

Quintile:	Low $\hat{b}_{y-1}$	High $\hat{b}_{y-1}$	ΔT-stat
T8: Net of Risk-free	+0.6%	+0.7%	(0.6)
T9: **LV** Net of Rf	+0.6%	+0.9%	(2.3)
T10: LV **Size/B-M** Adj	–0.3%	+0.2%	(3.3)

(LV= Low Vltlty. EW Quintiles. Excess= TB. 1963-2001)

Our Near Replication

$\hat{b}_{y-1}$-Spread Zero Pfio. Time-Series Regs. %/mo.

	ACXT8	ACXT9	ACXT10
ACX Alpha	0.11	0.23	0.44
(ACX T-stat)	(0.60)	(2.31)	(3.36)
			SMB HML
Sample:	All	LV	LV
Replication	**0.11**	**0.30**	**0.50**
(T-stat)	(0.60)	(1.85)	(3.37)

(Small differences in LV classification and SMB/HML adjustments.)

Placebo—Plain "All-Days" Beta

$\hat{b}_{\bar{y}}$-Spread Zero Pfio. Time-Series Regs. %/mo.

Similar to:	ACXT8	ACXT9	ACXT10
ACX Alpha		n/a	
(ACX T-stat)		n/a	
			SMB HML
Sample		LV	LV
LW Alpha	**0.03**	**0.20**	**0.45**
(T-stat)	(0.15)	(1.08)	(2.63)

Placebo is a little worse, but really quite similar!

So, what, if anything, is wrong here?

Average XMKT/mo in ACX sample: 0.54%/mo:

$$\Rightarrow \hat{b}_y \cdot \text{XMKT} \approx 0.77 \cdot 0.54\% \approx 0.42\%/\text{mo}$$

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$$\Rightarrow \hat{b}_y \cdot \text{XMKT} \approx 0.77 \cdot 0.54\% \approx 0.42\%/\text{mo}$$

Time-Series (FF) Regs, $\hat{b}_{y-1}$-Sort

	ACX T8	ACX T9	ACX T10	N/A
ACX Alpha	0.11	0.23	0.44	n/a
(T-stat)	(0.60)	(2.31)	(3.36)	n/a
				XMKT
			SMB	SMB
			HML	HML
Sample		LV	LV	LV
LW Alpha	0.11	0.30	0.50	**0.04**
T-stat	(0.60)	(1.85)	(3.37)	**(0.31)**

Is Exposure Alpha?

- Go long stocks with high X exposure
  Go short stocks with low X exposure
  - X can be a zero-investment currency pfio, or commodity pfio, or whatever.
- Look at a sample period in which $\bar{X} \gg 0$.

$\Rightarrow$ Portfolio should have pos avg rates of return.

- Average statement (not tautology).
- ACX looked at high-(down-)beta portfolios in a time of good stock-market performance.

Does FM Slope Imply FF Alpha?

- The 1-Factor CAPM model gives a prescription for how much pfio should have gone up.
  - FM Slope=Necessary, but not sufficient for FF Alpha.
- **In ACX, high-(down) beta pfios had higher rates of return only w/o XMKT control.**
- High-beta stocks ↑ more when/because market ↑.
- ...as they should have, given that they had positive exposures and the market went up,
- ...but high (down-)beta stocks did not even go up enough to “break even” in a “positive alpha” way.

What About Ex-Post Downbeta?

(ACX Fama-Macbeth Focus. Needed for Strong Positive.)

- We already know:
  - Down-betas $\approx$ Plain all-days betas.
  - From 1963-01, $\hat{b}_y \xrightarrow{+} r$ was good.
  - **Marginal** FM $\hat{\bar{b}}_y \to r$ was small 0.03.
  - Downbeta should be a little more positive in FF regs.
- So, was the marginal **realized simultaneous (ex-post)** $\hat{\bar{b}}_y$ predicted return even strong enough just to meet the 1-factor benchmark?

What About **Ex-Post** Downbeta?

	ACXT8	ACXT9	ACXT10	N/A
ACX Alpha		.....n/a		
(T-stat)		.....n/a		
				XMKT
			SMB	SMB
			HML	HML
Sample		LV	LV	LV
LW Alpha	0.14	0.25	0.45	–0.89
T-stat	(0.63)	(1.33)	(2.67)	(–0.78)

FM Reassessment

- Yes, there was a positive FM association between **ex-post** down-betas and rates of return;
- ...but it was not enough merely to beat the 1-factor target benchmark.

But it's 2016 now. What is the best inference today?

FM Reassessment

- Yes, there was a positive FM association between **ex-post** down-betas and rates of return;
- ...but it was not enough merely to beat the 1-factor target benchmark.

But it's 2016 now. What is the best inference today?

And in 2016? (Ex-ante $\hat{b}_y$)

Spec	ACXT8	ACXT9	ACXT10	N/A
ACX Alpha				
(T-stat)				
				XMKT
			SMB	SMB
			HML	HML
Sample		LV	LV	LV
LW Alpha	–0.28	–0.02	–0.02	–0.44
T-stat	(–1.32)	(–0.11)	(–0.12)	(–4.27)

Time-Series (FF) Regs, $\hat{b}^{-}_{y-1}$

From 1963–2016:

- Higher $\hat{b}^{-}_{y-1}$ stocks did not even have higher average rates of return;
- ...but XMKT continued to be very positive;
- ...thus 1-F alpha of $\hat{b}^{-}_{y-1}$ was not just not positive, it was negative;
- ...just as it is for $\hat{b}_{y-1}$ in Frazzini-Pedersen.

Did Down-Beta $\hat{b}^-$ Give Pos Alpha?

Relative to what?

- Risk-Neutral Model?
  A: Yes, as of 2001.
  A: No, as of 2016.
- CAPM? A: Never.
- Fama-French 3F Model? A: Never.
- (Fama-French 5F+UMD Model? A: Never.)
- down-beta roughly similar to plain beta, never offering extra.

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- To test a beta-risk-reward argument,
- do not form zero-investment test portfolio on the basis of difference of

$$\hat{b}_{\bar{y}} - \hat{b}_y$$

- ...unless you want to learn whether $\hat{b}_{\bar{y}}$ has a less negative relation with future stock returns than $\hat{b}_y$!
- ...which would be sort of silly as an AP test whether investors need comp for (down-)beta risk
- ...which is sort of the case in the ACX GMM spec, too.

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4. GMM on 25 FF pfios (T6)

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- Down-beta helps explain 25 FF portfolio returns.
  - remarkable, given motivation about pfio info destruction.
- ...but with the wrong sign ?!? b_m is coef on r_m.

	a	b_m	b_{m^-}
T6 Spec II	1.35	–17.73	22.84
$E(I(x) \cdot r) = 0$	[8.70]	[3.03]	[2.16]

- ...and see warning on prev page.

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We need to learn about down-beta, not win an argument.

We need to learn what we have missed.

We could not get a hold of ACX, so apologies for not considering and investigating more counterarguments.

Hopefully, we will soon improve paper with Andrew's comments. We want to end up with a better synthesis than his thesis and our antithesis.

...and of course, the **Critical Finance Review** is very interested in this kind of exchange between critique and authors.

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Part 3: Down-Beta in Asset Classes

- Lettau-Maggiori-Weber (2014).
- Uses **full-sample** betas, not realized betas.
- Like every paper, makes some choices. All ok.

- Common misconception, already nicely noted in LMW: Currencies are mostly just completely unrelated investments...like cash.

Ex-Ante vs Full-Window Betas

- Full-Window betas may be better than ex-ante,
- ...esp because we have low power on down-market classification.

- **Ex-Ante** Down-Beta Inference in FM:
  - some results become weaker (a few become stronger).
  - LMW's results do not generally reverse, unlike ACX's.

(sovereign bonds may become more interesting with more data.)

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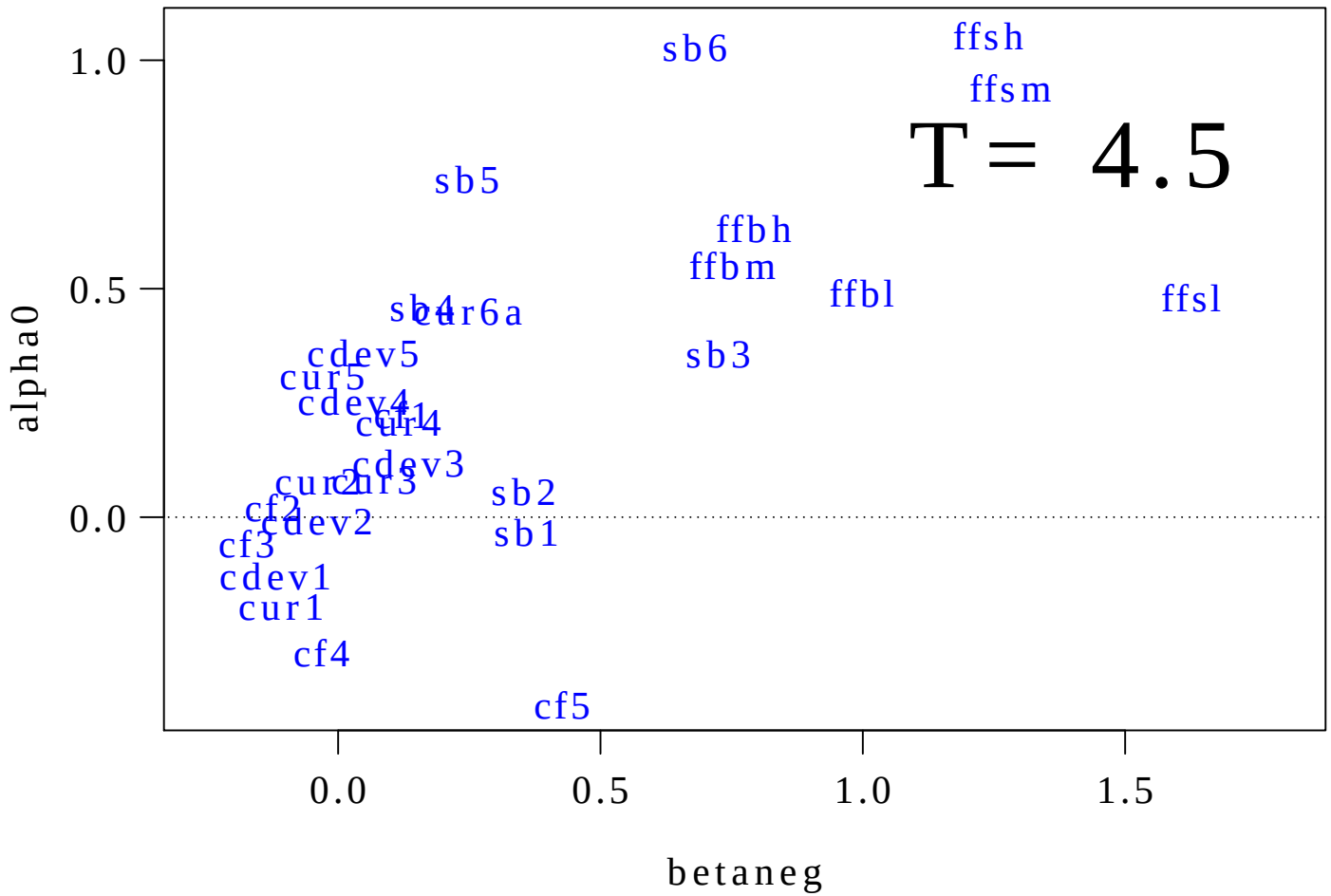
Can CAPM or FFM explain Alphas? Is Downbeta helpful?

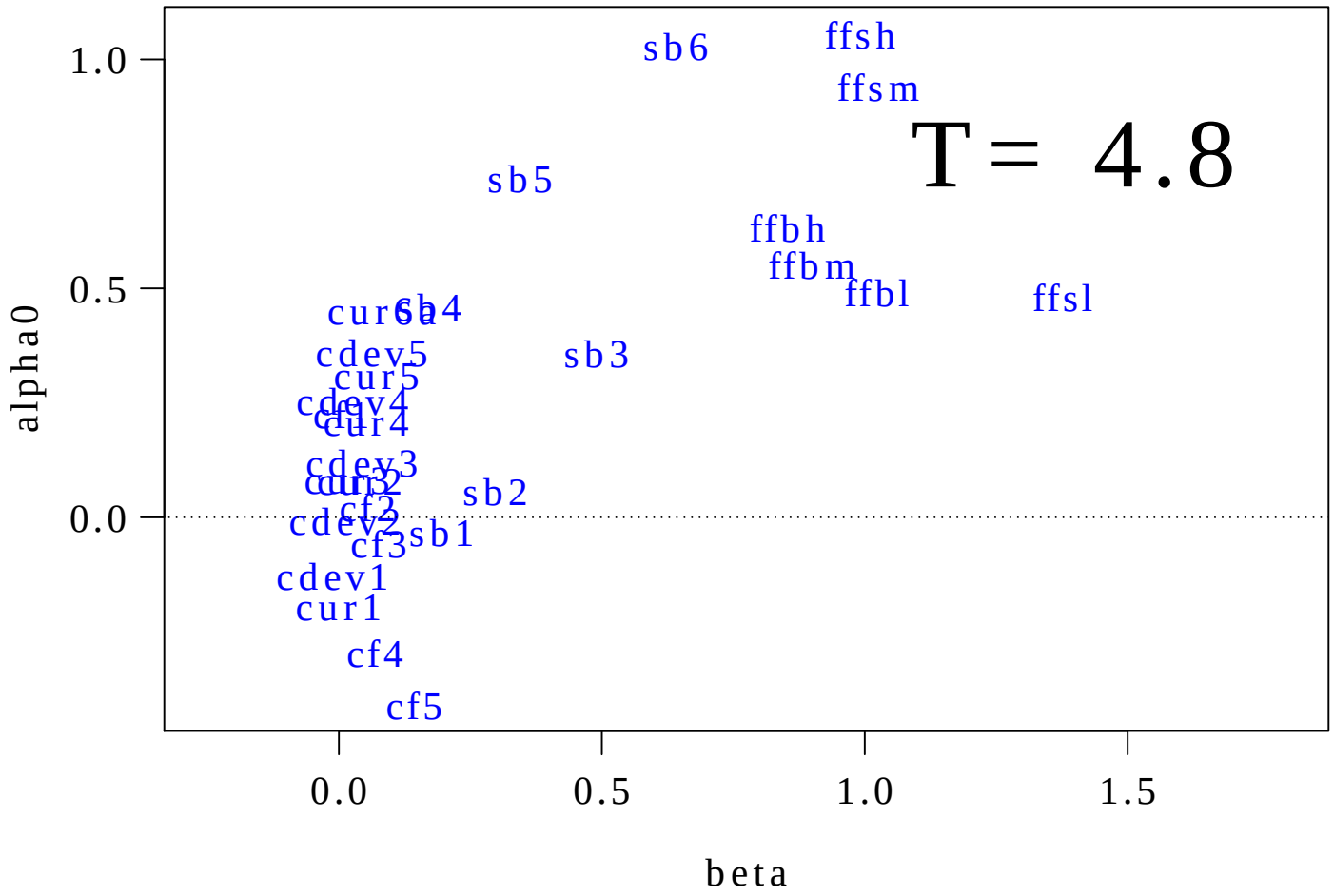
	Down--Beta	All-Days Beta	Diff-erence
Rf	$\hat{b}^{-} \rightarrow \alpha_{0F}$	$\hat{b} \rightarrow \alpha_{0F}$	$\hat{b}_y^{-} - \hat{b} \rightarrow \alpha_{0F}$
~~**CAPM**~~	~~$\hat{b}^{-} \rightarrow \alpha_{1F}$~~	~~$\hat{b} \rightarrow \alpha_{1F}$~~	~~$\hat{b}_y^{-} - \hat{b} \rightarrow \alpha_{1F}$~~
FFM	$\hat{b}^{-} \rightarrow \alpha_{3F}$	$\hat{b} \rightarrow \alpha_{3F}$	$\hat{b}_y^{-} - \hat{b} \rightarrow \alpha_{3F}$

$\hat{b}^{-} \longrightarrow \alpha_{0F}$



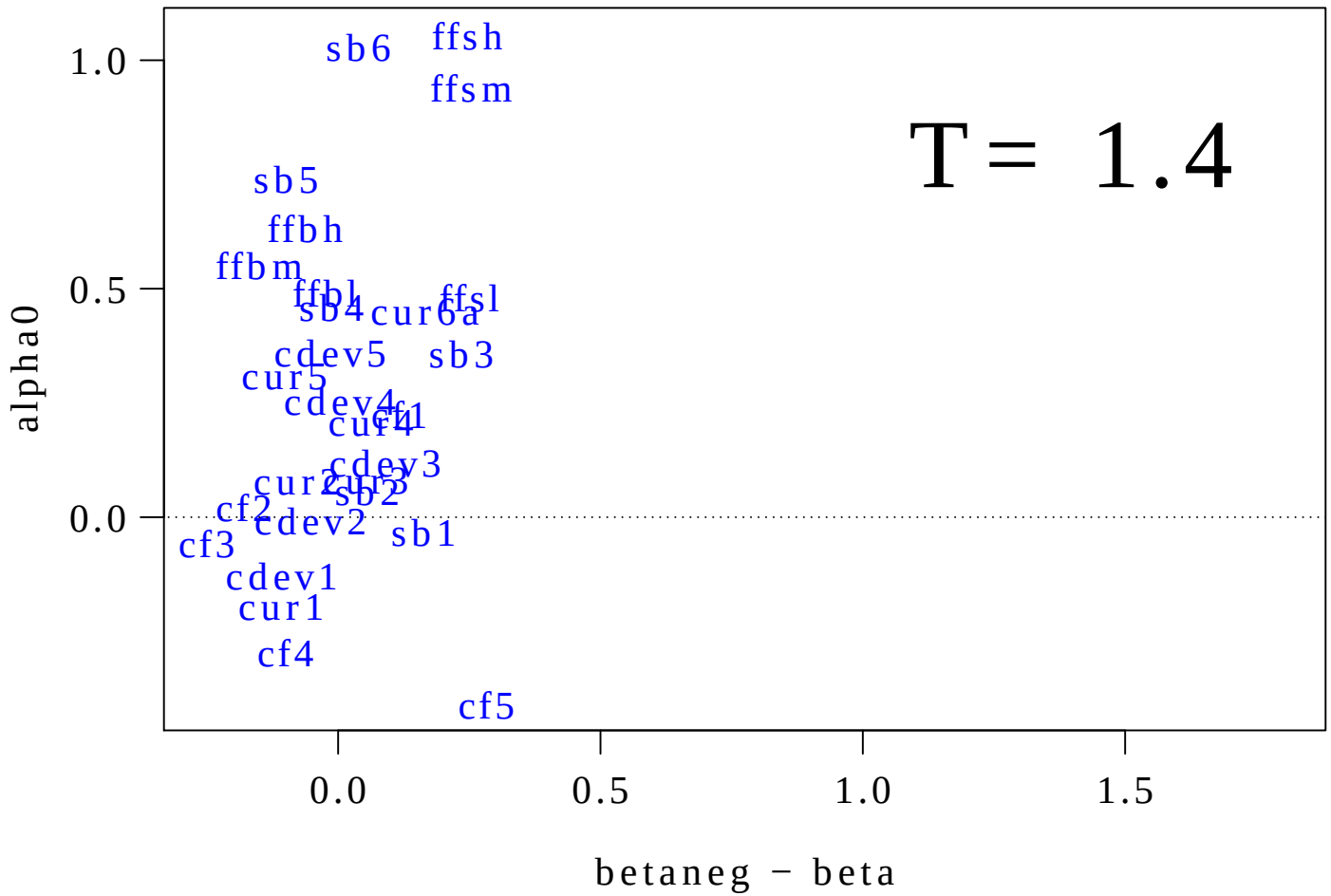
Positive between downbeta and risk-free adj returns.

$$\hat{b} \longrightarrow \alpha_{0F}$$



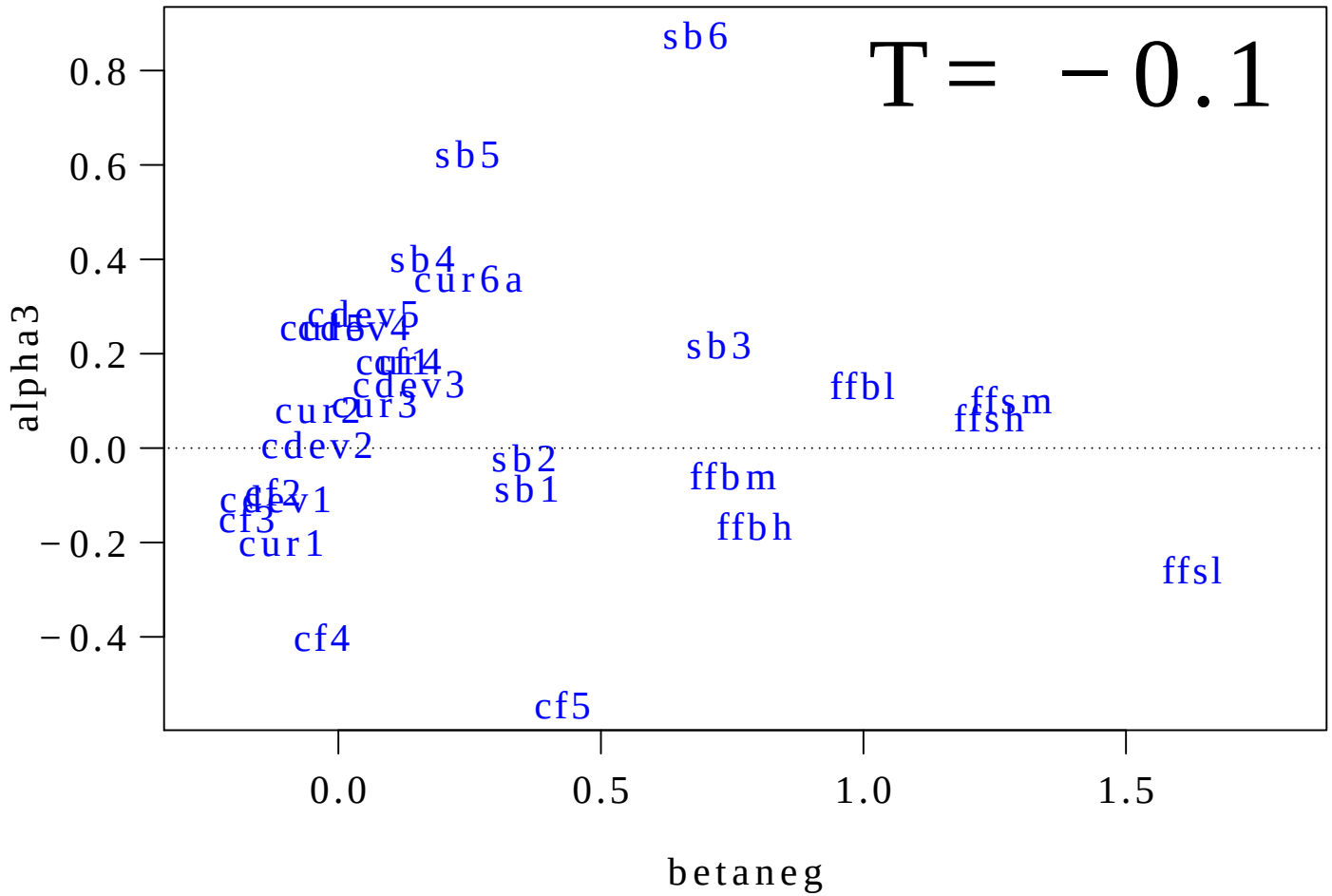
Positive between **plain** beta and risk-free adj returns.

$$\hat{b}^{-} - \hat{b} \longrightarrow \alpha_{0F}$$



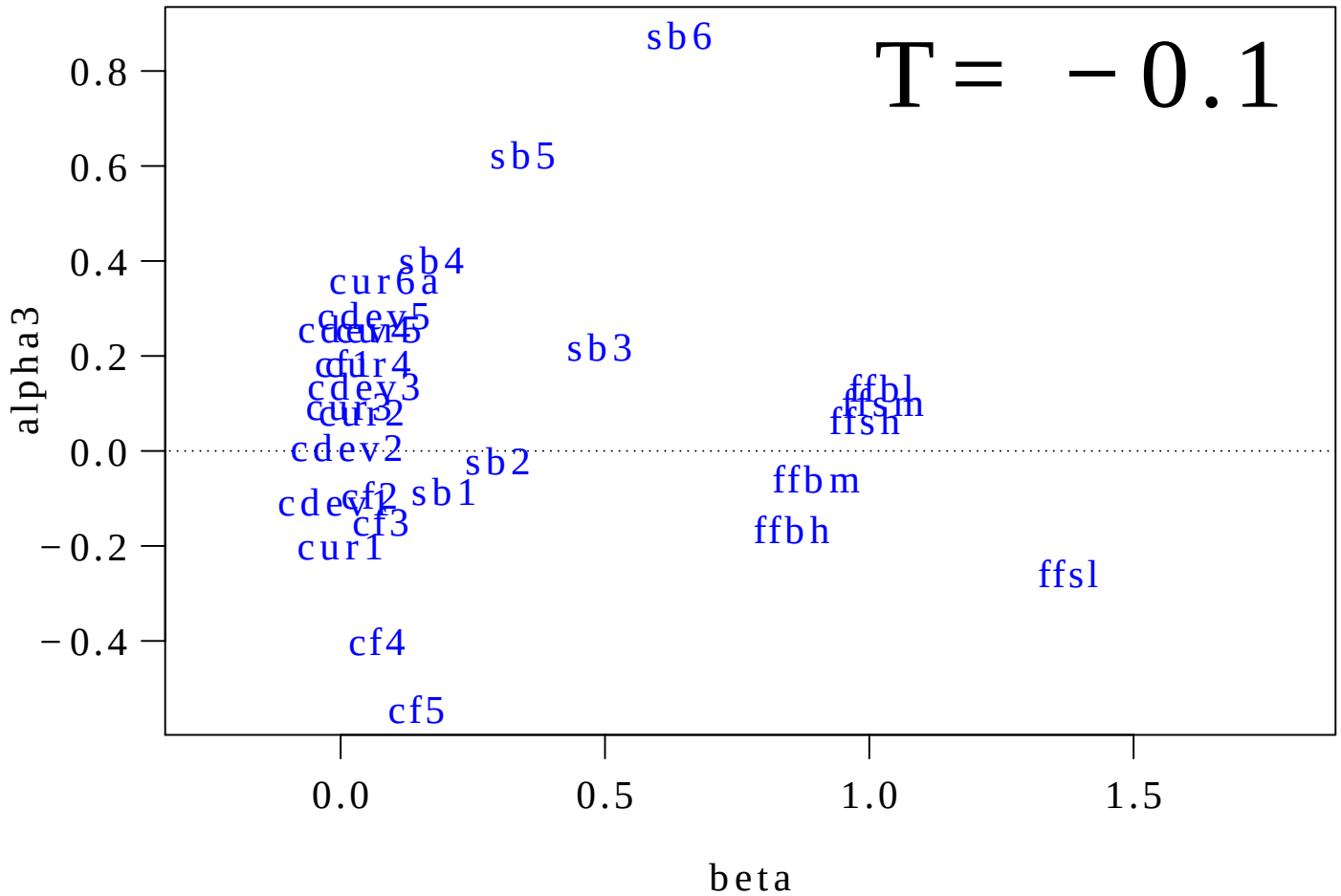
Positive between **delta beta** and risk-free adj returns.

$\hat{b}^- \longrightarrow \alpha_{3F}$



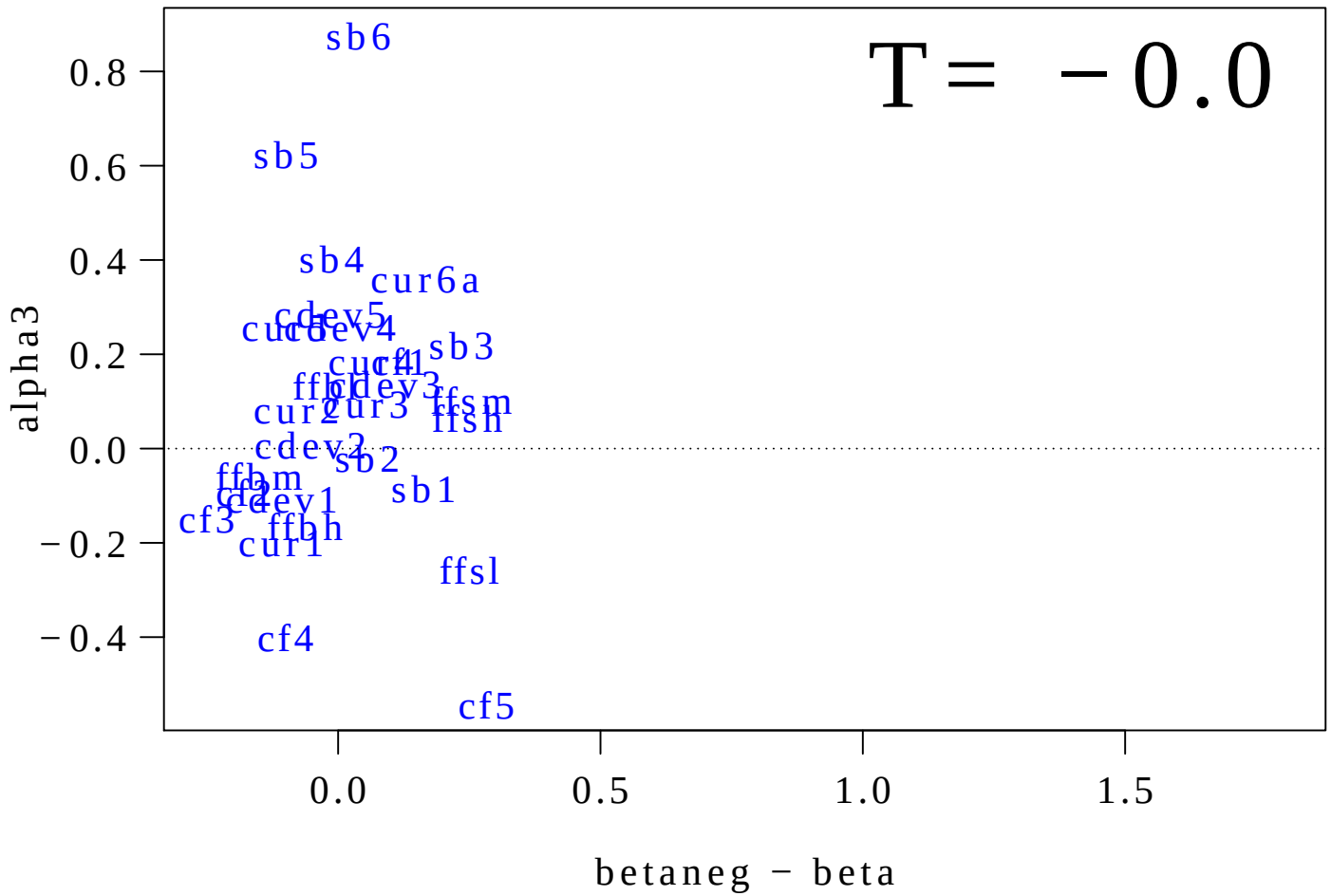
No association between down-beta and **FFM**-adj.

$\hat{b} \longrightarrow \alpha_{3F}$



No association between **plain** beta and FFM-adj.

$$\hat{b}^{-} - \hat{b} \longrightarrow \alpha_{0F}$$



No association between **beta-diff** and FFM-adj.

Summary on Beta Prediction

- Plain all-days daily-return betas work great for down-markets, too.
- Est'd **ex-ante** down-betas are useless:
  - Even if you care only about down-beta
  - You are still better off using all-days daily returns.

Summary on Return Prediction

Despite positive ***Fama-Macbeth*** *coefficients for* ***ex-post*** *down-betas associating with stock returns:*

- For many investment strategies, differences between FM and FF tests are modest
  - but not in near-beta-related strategies,
  - where strategy has to beat market premium $ER_m - r_f$.
- Down-beta-sorted pfios, ex-ante or ex-post, have zero or negative **CAPM/FFM alphas**.
  - $\hat{b}_y^-$ are primarily just (noisier) proxies for $\hat{b}_y$.
  - $\hat{b}_y^-$ do not help resolve asset-pricing puzzles.
  - Returns were not unusual on down-beta dimension.

A Better Market-Beta Estimator

(brand-new, 1 week old.)

performance metric

I will predict

- future ols(/other) market-beta **estimates**
- never future average returns.

best beta estimator known to-date

- daily stock returns
- about 1-3 years of data.
- vasicek and its derivatives
  - (random-effects and/or bayesian justification if no drift.)
  - Levi-Welch linear de-bias.

more alternatives below.

vasicek disadvantages

- optimal design was never suited to problem:
  - designed for measurement error,
  - not for underlying beta drift
  - (ergo 12–24 months windows)
- good R^2, but badly biased
  - levi-welch (2017) suggests empirical de-biasing
  - requires another stage
- spooky entangled estimates
- requires multi-step ts and xs procedure
- I will show you a better and simpler estimator

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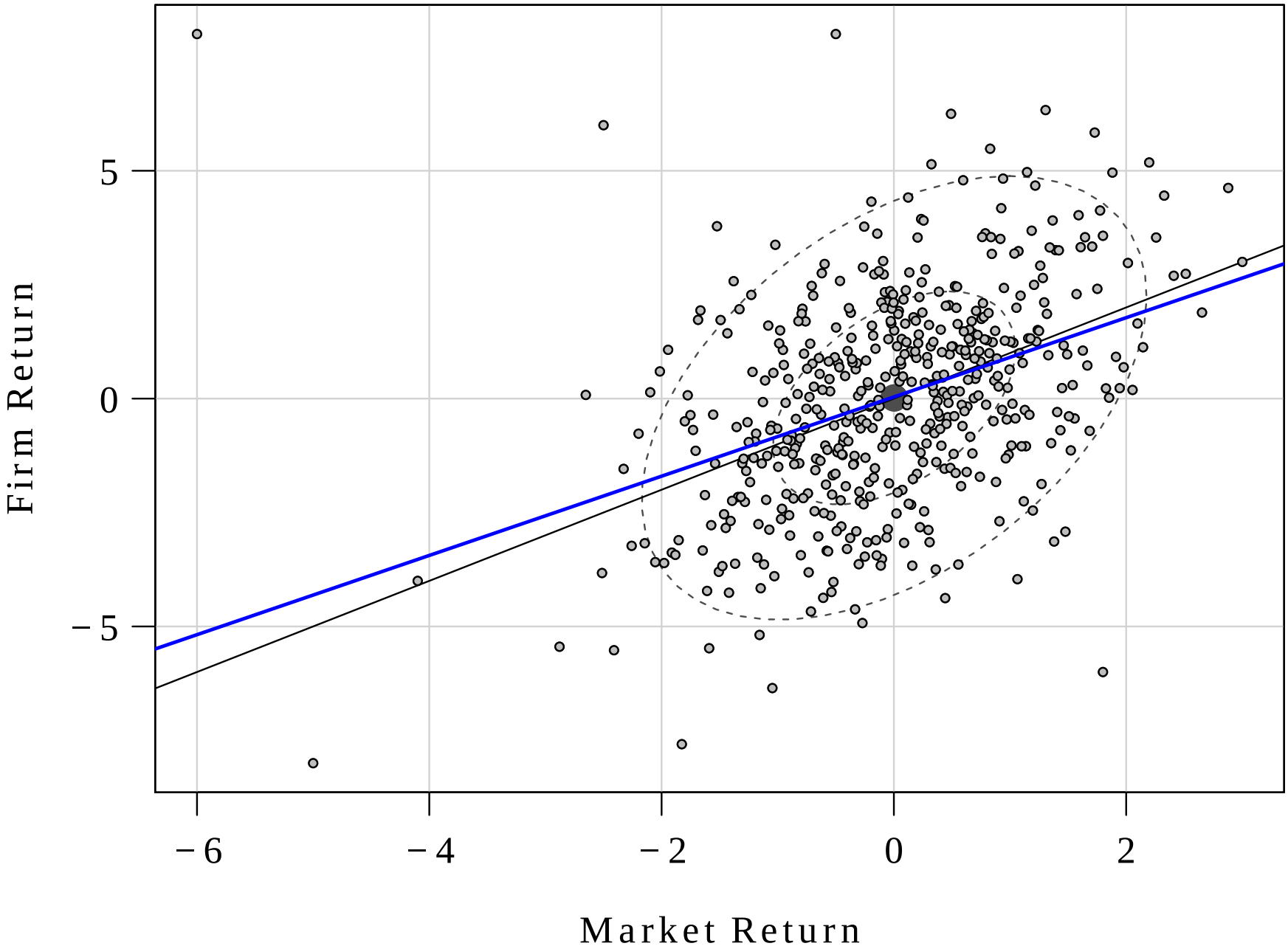
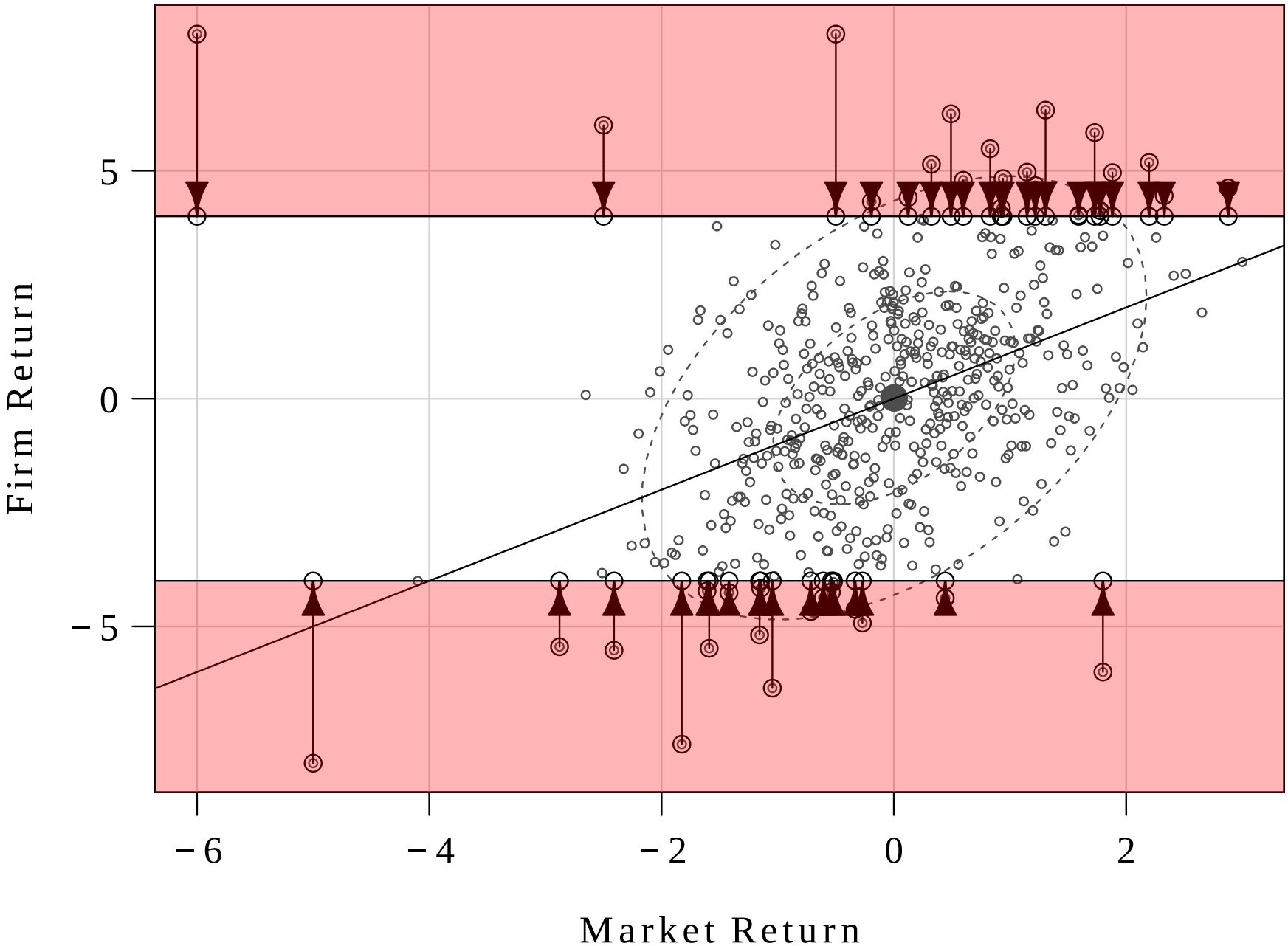

Firm Return
5
0
−5
−6
−4
−2
0
2
Market Return


Firm Return
5
0
−5
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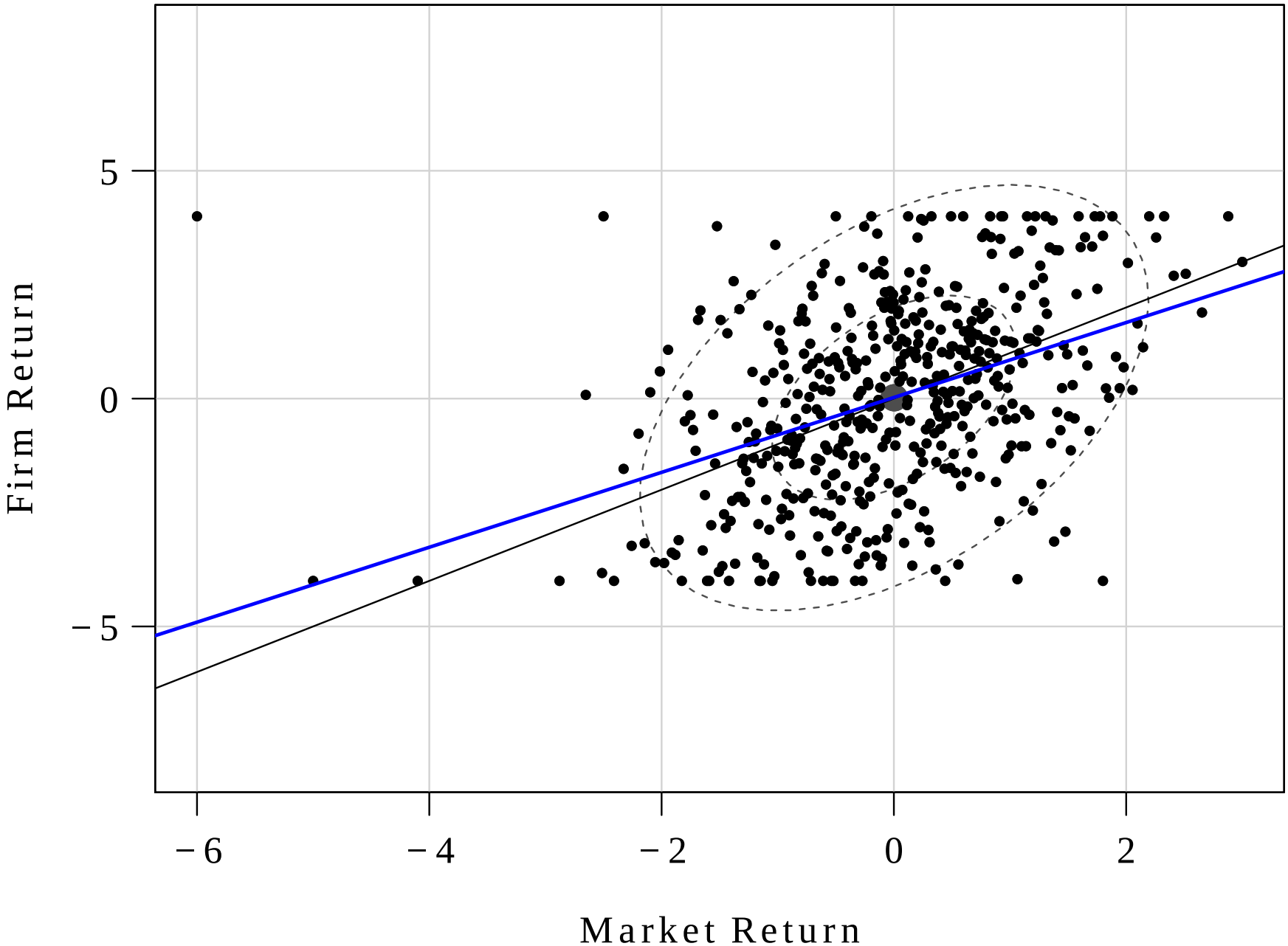
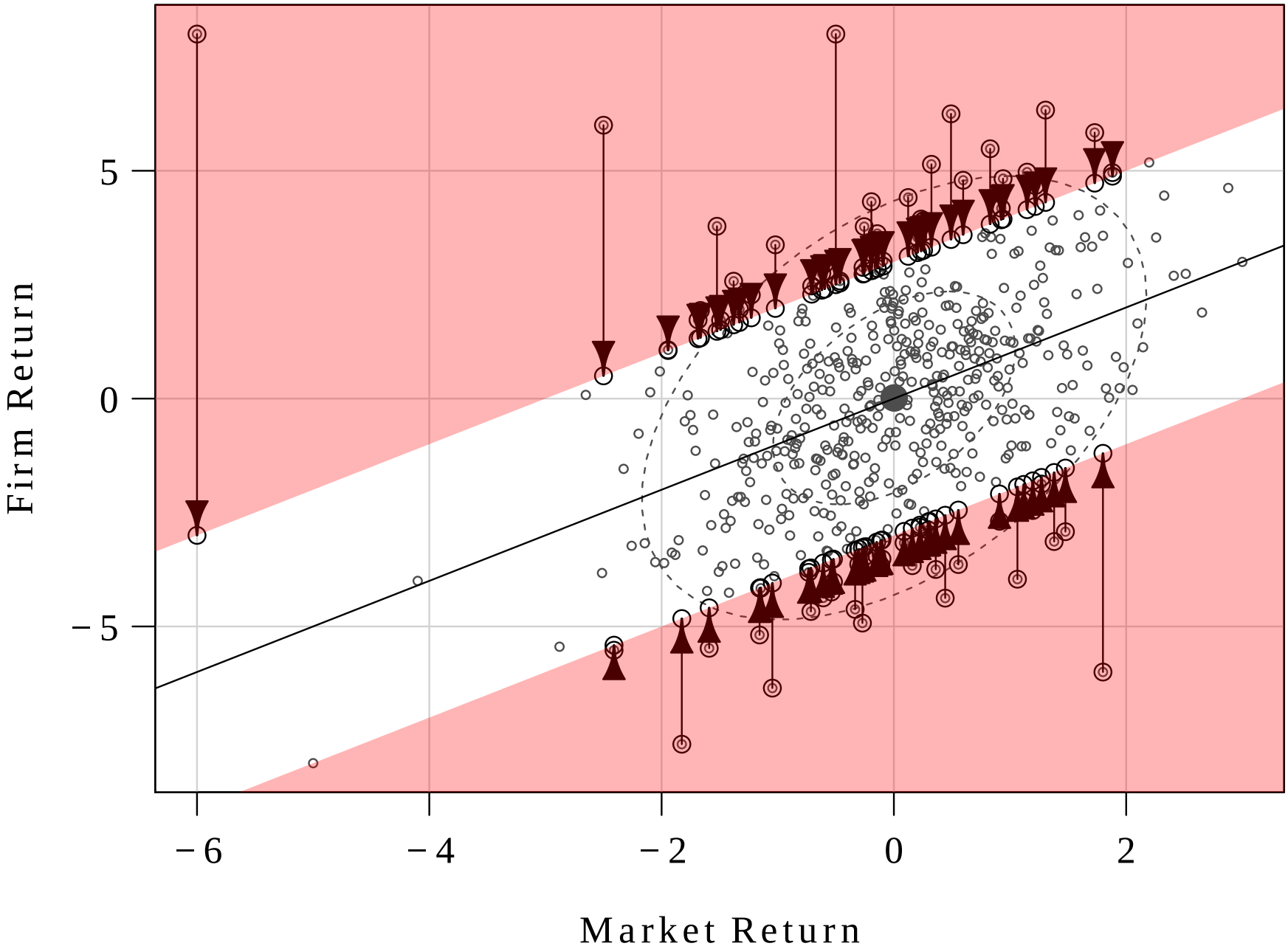
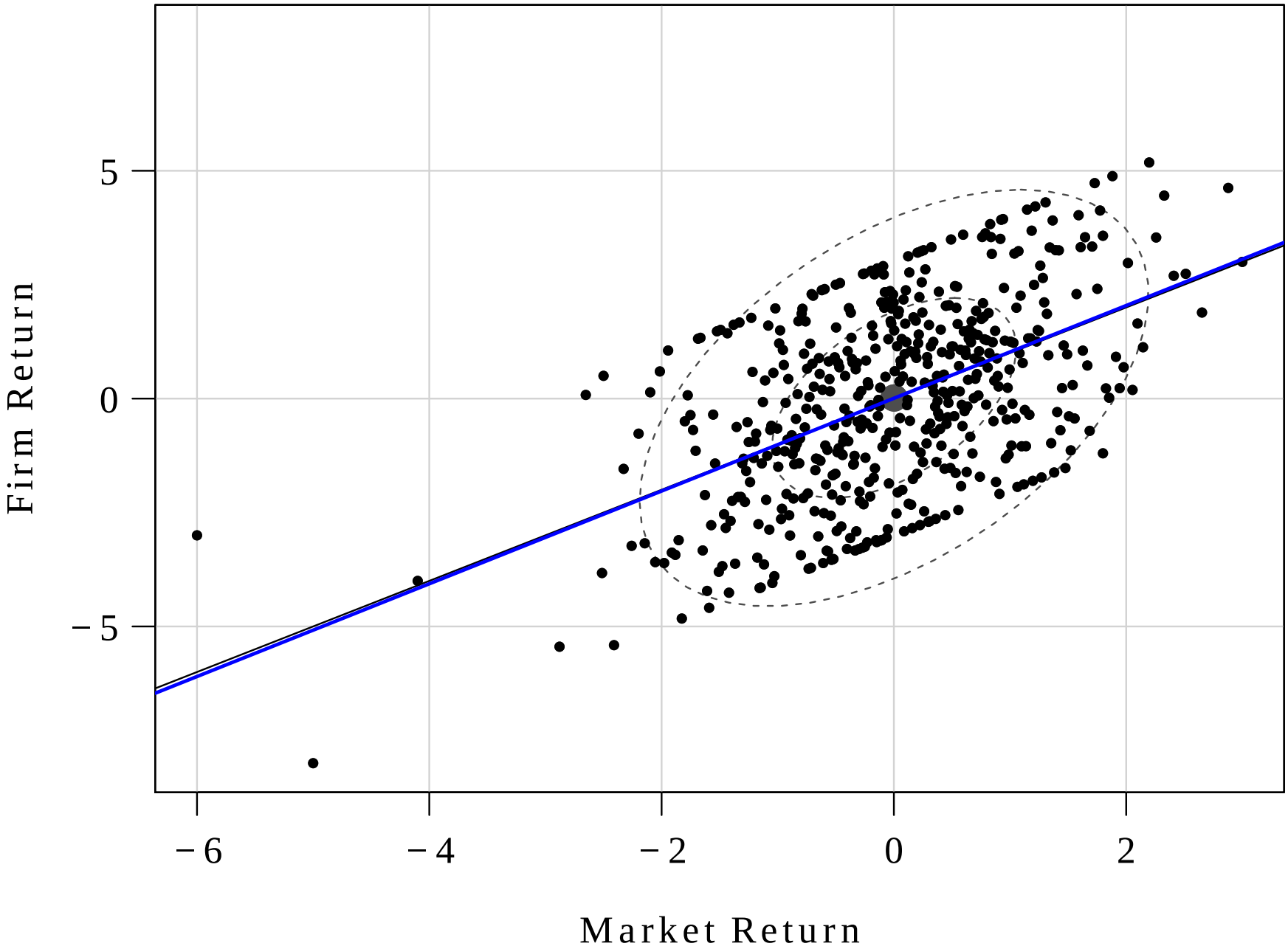
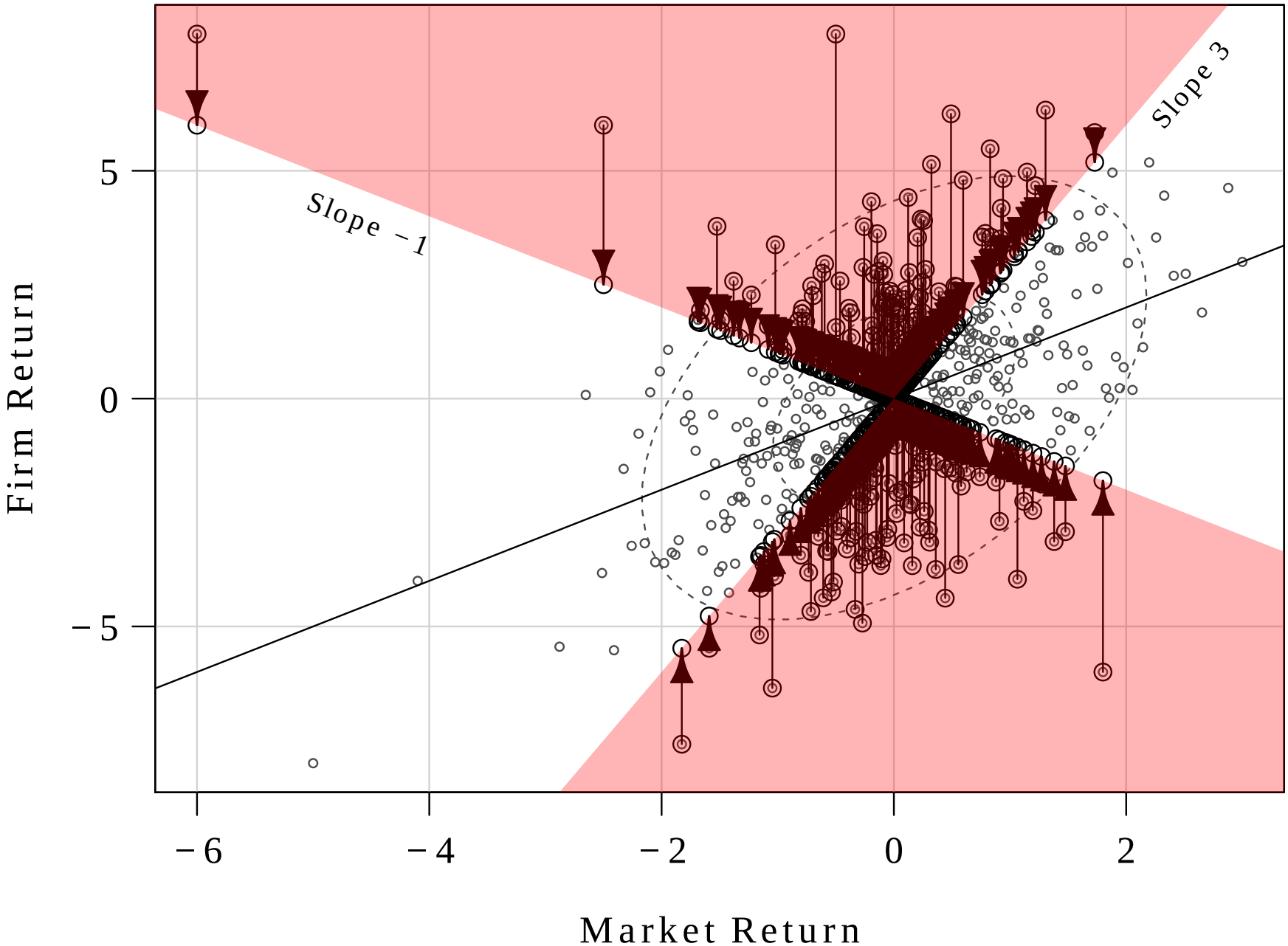
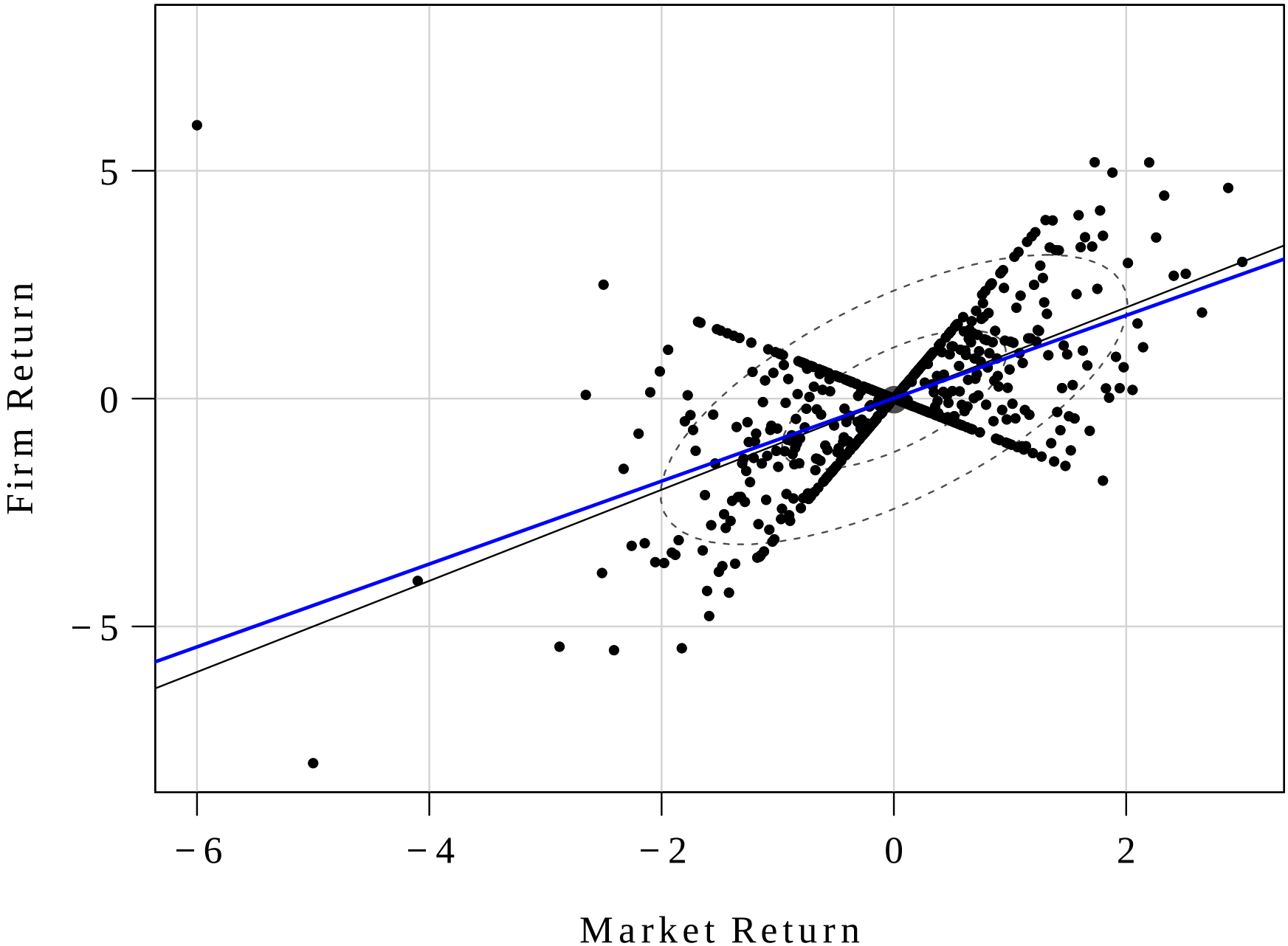
2
Market Return


Firm Return
5
0
−5
−6
−4
−2
0
2
Market Return


Firm Return
5
0
−5
−6
−4
−2
0
2
Market Return


Firm Return
5
0
−5
−6
−4
−2
0
2
Market Return


Slope −1
Slope 3
5
0
−5
−6
−4
−2
0
2
Firm Return
Market Return


Firm Return
5
0
−5
−6
−4
−2
0
2
Market Return

beta slope winsorized (bsw)

1. 12–24 mos of **daily** stock returns
2. winsorize all returns ($\Delta_s = 2$):

$$\mathrm{rsw}_{i,t} \in 1.0 + \left[-\Delta_s, \Delta_s\right] \cdot r_{m,t} .$$

3. estimate ols market-model

$$\mathrm{rsw}_{i,t} = a_i + \mathrm{bsw}_i \cdot r_{m,t}$$

(just a reuse of the model with a reasonable prior. note: model-specific.)

why $\Delta_s = 2$?

- fewer than 1% of betas exceed –1 and +3
- fewer than 0.03% repeat in consecutive years
- beyond, no monotonicity between b_t and $E(b_{t+1})$
- not philosophical, but also not highly searched:
  - you could also use [–0.5, 2.5] or [–3, 5].
  - lower Δ_s forces too much towards 1.
  - higher Δ_s forces nada.

does it matter?

are betas even different?

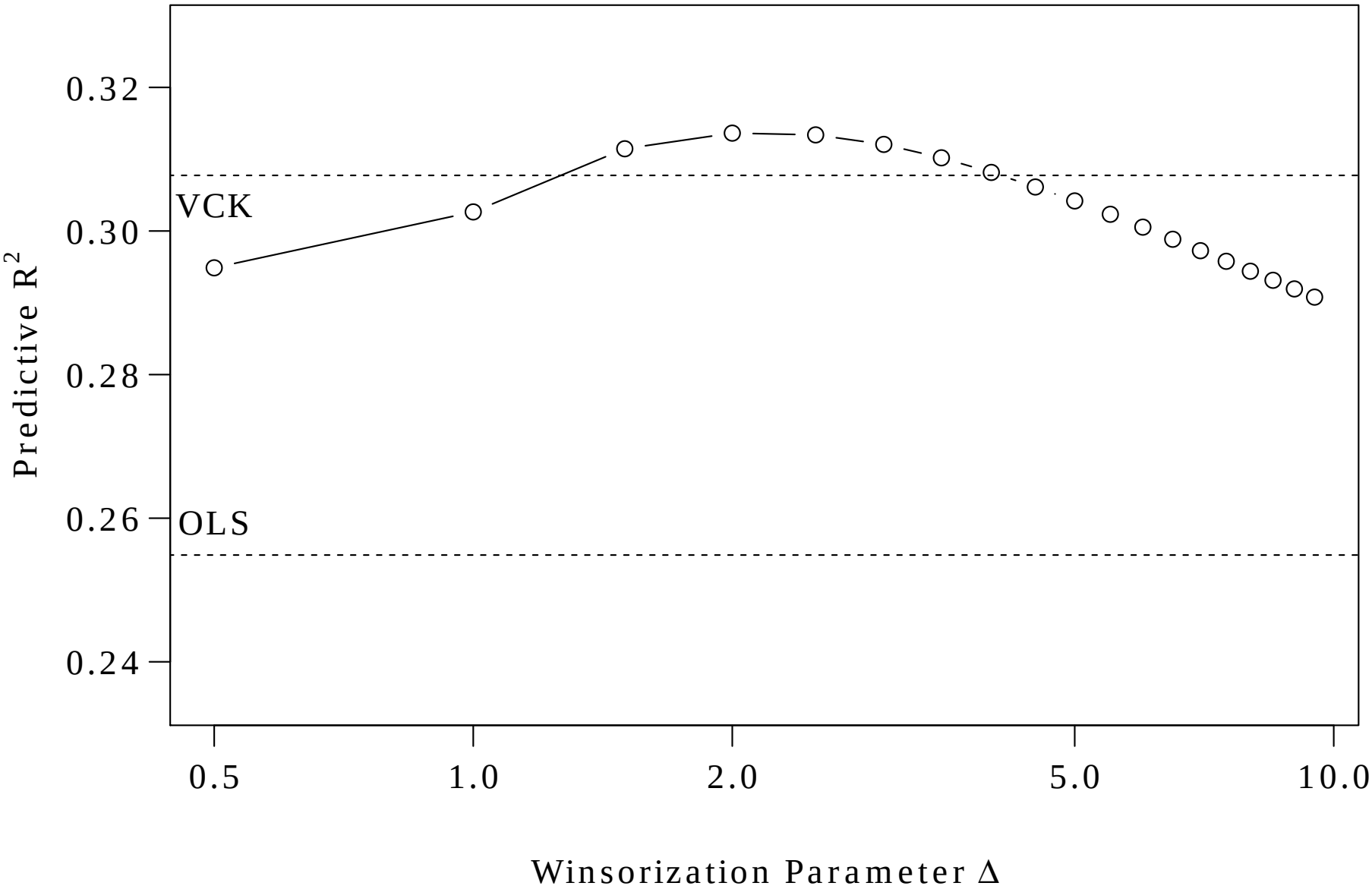
$$\text{rmsd}\ (\ \text{bols}_D\ ,\ \text{bsw}\ ) \approx 0.37$$

$$\text{rmsd}\ (\ \text{bvck}_D\ ,\ \text{bsw}\ ) \approx 0.20$$

$$\text{rmsd}\ (\ \text{bols}_M\ ,\ \text{bsw}\ ) \approx 0.60$$

"gamma" panel reg for $bols_{t+1}$

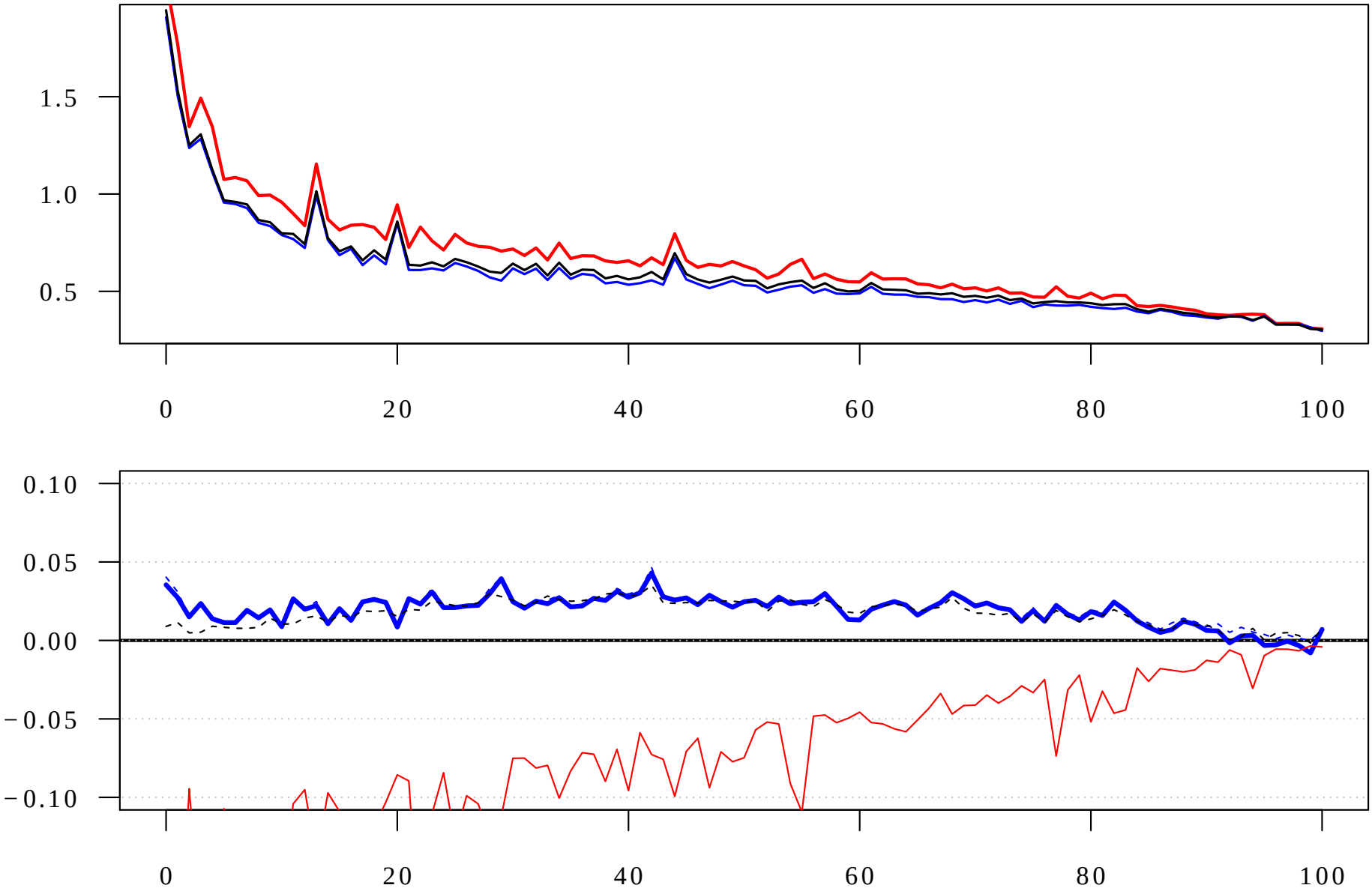
	γ_0	se(γ_0)	γ_1	se(γ_1)	R^2
(bols)	0.34	.004	0.54	.005	25.5%
(bvck)	0.19	.002	0.74	.002	30.8%
... (blw)	–0.01	.003	0.98	.003	same
level (blw)	0.27	.002	0.70	.003	29.7%
band (bbw)	0.04	.002	0.93	.003	30.9%
slope (bsw)	0.01	.002	0.96	.003	**31.4%**
slope + v	–0.01	.003	1.00	.003	31.5%


0.32
0.30
0.28
0.26
0.24
VCK
OLS
Predictive R^2
0.5
1.0
2.0
5.0
10.0
Winsorization Parameter Δ

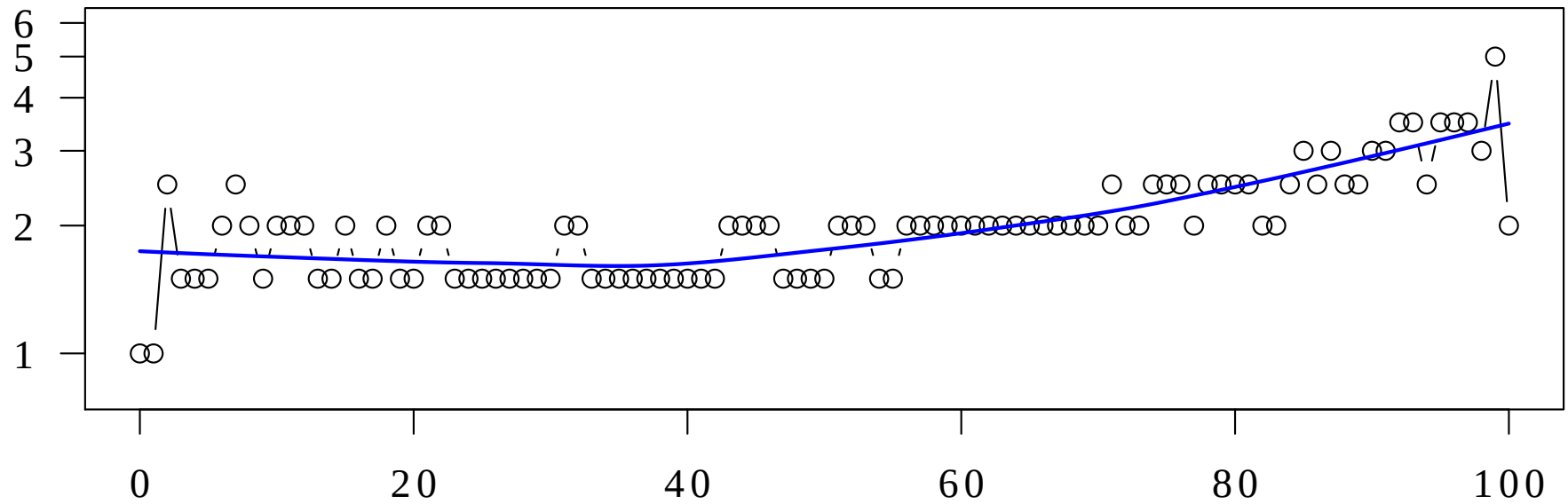
nothing edgy

- very stable by year.
- very stable by ols beta.
- no meaningful improvement by varying Δ_s.
  - even by own lagged beta, beta-sd, marketcap, trading volume, volatility, etc.
  - first-stage firm-specific estimated deltas don't help much. will show you best.

rmse by market cap percentile



Δ^* by market cap percentile



possible improvements: obtain mcap rank, then

- more winsorization ($\Delta_s = 1.5$) for small-caps (rank < 40%),
- less winsorization ($\Delta_s = 3$) for big-caps (rank > 80%).

another 2% R^2 improvement

steep exponential decline ($\approx \exp[-2\Delta d/252]$)

	Now	3 mo	6mo	1yr	2yr
WLS weights	1.0	80%	50%	10%	2%

- WLS allowed for kink.
- no loss of observations.
- trivially easy in time.

another 1% R^2 improvement

- add one extra variable reflecting firm-size or dollar trading volume.
  - big firms have bigger market-betas (yes!),
  - but use requires first-stage regression,
  - and marketcap requires merging, data loss, etc.
- I could find no other useful accounting compustat or crsp derived variable or ratio.

monthly-overlaps + dimson + fp

	R^2 with x being only				
y ↓	self	bsw	vck	dim	fp
ols	38%	44%	43%	28%	27%
vck	50%	51%			
bsw	— 57% —		⇒ R^2 to β_{true} should be ≈ 75%		
dim	22%	30%	better use bsw if interested in dim		
fp	21%	30%			

→ what should you use if you care (but why?) about future dimson or fp estimates?

monthly-frequency return data?

- even long-window monthly betas are miserable predictors of anything (like R^2 of $< 15\%$, not 40%).
- daily predicts monthly better than monthly itself.
- $\rightarrow$ use daily even if interested in monthly.

conclusion

- novel slope winsorization method afaik.
- novel application of winsorization method in important context of market-beta estimation.
- only simple use of prior. no 1st stage.
- superb ease of use. pto.

so why not?

```
beta <- function(...) coef(lm(...))[2]
wins.rel <- function( r, rmin, rmax ) {
   rl <- ifelse( (rmin<rmax), rmin, rmax )
   ru <- ifelse( (rmin<rmax), rmax, rmin )
   ifelse( r<rl, rl, ifelse(r>ru, ru, r) )
}

delta <- 2
wri <- wins.rel( ri, (1-delta)*rm, (1+delta)*rm )

bsw <- beta( wri ~ rm )

wbsw <- beta( wri ~ rm, w=exp(-2*(length(ri):1)/256) )
```